Code : 101202

B.Tech 2nd Semester Exam., 2021

(New Course)

MATHEMATICS—II

(Differential Equations)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are **NINE** questions in this paper.*

(iii) *Attempt **FIVE** questions in all.*

(iv) *Question No. **1** is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) Integrating factor of the differential equation $dy = \{e^{x-y}(e^x - e^y)\}\,dx$ is

(i) e^{e^x}

(ii) e

(iii) e^x

(iv) e^{2x}

(b) The particular integral (PI) of the differential equation

$$\frac{d^2y}{dx^2} + 4y = \cos 2x$$

is

(i) $\dfrac{\sin 2x}{2}$

(ii) $\dfrac{x \sin 2x}{2}$

(iii) $\dfrac{x \sin 2x}{4}$

(iv) $\dfrac{x \cos 2x}{2}$

(c) The differential equation whose auxiliary equation has the roots 0, −1, −1 is

(i) $\dfrac{d^3y}{dx^3} - 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$

(ii) $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$

(iii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$

(iv) $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + y = 0$

(d) If $J_0(x)$ and $J_1(x)$ are Bessel functions, then $J_1'(x)$ is given by

(i) $-J_0(x)$

(ii) $J_0(x)-\frac{1}{x}J_1(x)$

(iii) $J_0(x)+\frac{1}{x}J_1(x)$

(iv) $J_0(x)$

(e) The complete solution of the partial differential equation $p^2+q^2=2$ is

(i) $z=ax+\sqrt{2-a^2}y+c$

(ii) $z=ax-\sqrt{2-a^2}y+c$

(iii) Either (i) or (ii) true

(iv) (i) and (ii) both true

(f) The partial differential equation

$$5\frac{\partial^2 z}{\partial x^2}-5\left(\frac{\partial z}{\partial x}\right)^2+6\frac{\partial^2 z}{\partial y^2}x=xy$$

is classified as

(i) parabolic

(ii) hyperbolic

(iii) elliptic

(iv) None of the above

(g) The solution of the partial differential equation

$$x^3\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=0$$

if $u(0, y)=10e^{5/y}$ using method of separation of variable is

(i) $10e^{5/2x^2}e^{5/y}$

(ii) $10e^{-5/2y^2}e^{5/x}$

(iii) $10e^{-5/2y^2}e^{-5/x}$

(iv) $10e^{-5/2x^2}e^{5/y}$

(h) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function, then $f'(z) =$

(i) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

(ii) $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

(iii) $\frac{\partial v}{\partial y} - i\frac{\partial v}{\partial x}$

(iv) $\frac{\partial u}{\partial x} + i\frac{\partial u}{\partial y}$

(i) The value of $\int_C \frac{3z^2 + 7z + 1}{z + 1} dz$ where C is $|z| = 1$ is

(i) $2\pi i$

(ii) 0

(iii) πi

(iv) $\pi i/2$

(j) The value of $\Delta \tan^{-1} x$ is

(i) $\Delta \tan^{-1}\left(\frac{h}{1 + hx + x^2}\right)$

(ii) $\Delta \tan^{-1}\left(\frac{h + 2x}{1 + hx + x^2}\right)$

(iii) $\Delta \tan^{-1}\left(\frac{h - 2x}{1 + hx + x^2}\right)$

(iv) $\Delta \tan^{-1}\left(\frac{h + 2x}{1 - hx - x^2}\right)$

2. Solve the following differential equations :

(a) $2ydx + x(2\log x - y)dy = 0$ 7

(b) $y - 2px = \tan^{-1}(xp^2)$ 7

3. (a) Use the method of variation of parameters to find the solution of the given differential equation

$$y'' - 2y' + y = e^x \log x$$ 6

(b) Find the series solution of the differential equation

$$(x^2 - 2x + 1)\frac{d^2y}{dx^2} + (4x - 4)\frac{dy}{dx} + (x^2 - 2x + 3)y = 0$$

about the point $x = 1$. 8

4. Prove the following :

(a) $\int J_3(x)dx = c - J_2(x) - \frac{2}{x}J_1(x)$, where c is arbitrary constant. 6

(b) $$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$ 8

5. (a) Solve the following differential equation : 7

$$x^2(y - z)p + y^2(z - x)q = z^2(x - y)$$

(b) Find the complete integral of

$$(p^2 + q^2)y = zq$$ 7

6. (a) Prove that the general solution of the partial differential equation

$$[D - mD' - a]^2 z = 0$$

where

$$D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y}$$

is given as

$$z = e^{ax} f_1(y + mx) + xe^{ax} f_2(y + mx)$$ 8

(b) Find the general solution of the given partial differential equation

$$(D^3 D'^2 + D'^3 D^2 - 5D^2 D'^2 - 2D^3 D' + 6D^2 D')z = e^{x-y}$$ 6

7. Find the D'Alembert's solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$ 14

Or

(a) The following values of the function $f(x) = \sin x + \cos x$, are given

x	:	10°	20°	30°
$f(x)$	:	1·1585	1·2817	1·366

Construct the quadratic Lagrange interpolating polynomial that fits the data. Hence, find $f(\pi/12)$. Compare with the exact value. 7

(b) Derive the Newton's forward difference formula using the operator relations. 7

8. Find the solution of the differential equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

subject to conditions—

(i) u is not infinite at $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = \ell$

(iii) $u = \ell x - x^2$ for $t = 0$ between $x = 0$ and $x = \ell$. 14

Or

(a) Check whether the function $f(z)$ defined by

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is continuous and the Cauchy-Reimann equations are satisfied at the origin or not. Also check whether the function $f(z)$ defined above is analytic at $z = 0$ or not. 7

(b) Find the image of the infinite strip

$$\frac{1}{4} \leq y \leq \frac{1}{2}$$

under the transformation $w = \dfrac{1}{z+c}$, where c is a real constant. 7

9. Describe the Runge-Kutta method of fourth order for the solution of initial value problem. Given the initial value problem $y' = 1 + y^2$, $y(0) = 0$. Find $y(0 \cdot 6)$ by using Runge-Kutta method of fourth order by taking $h = 0{\cdot}2$. 14

Or

(a) Find the sum of the residues of the function

$$f(z) = \frac{\sin z}{z \cos z}$$

at its poles inside the circle $|z| = 2$. 7

(b) Evaluate the integral

$$\oint_C \frac{3z^2 + z}{z^2 - 1} dz$$

where C is the circle $|z - 1| = 1$. 7

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