# Code : 105102

## B.Tech 1st Semester Exam., 2018 (New)

MATHEMATICS—I

**(Calculus and Linear Algebra)**

*Time* : 3 hours *Full Marks* : 70

*Instructions* :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are* **NINE** *questions in this paper.*

(iii) *Attempt* **FIVE** *questions in all.*

(iv) *Question No.* **1** *is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) If $\varphi(x, y, z) = 3x^2y - y^3z^2$, then $\nabla\varphi$ at the point $(1, 1, -2)$ is

(i) $12i - 9j - 16k$

(ii) $6i - 9j + 4k$

(iii) $6i - 9j - 4k$

(iv) $-12i - 9j + 16k$





(b) If $a < b$, then $\int_a^b |(x-a)+(x-b)|dx$ is equal to

(i) $\frac{(b-a)^2}{2}$

(ii) $\frac{(b^2-a^2)}{2}$

(iii) $\frac{(a^3-b^3)}{2}$

(iv) $(b-a)^2$

(c) The value of $\iint x^2y^2 dxdy$ over the region $x^2+y^2 \le 1$ is

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{12}$

(iii) $\frac{\pi}{24}$ (iv) $\frac{\pi}{48}$

(d) If $A(2) = 2i - j + 2k$, $A(3) = 4i - 2j + 3k$, then $\int_2^3 A.\frac{dA}{dt}dt$ is

(i) 5 (ii) 10

(iii) 15 (iv) 20

(e) The Fourier series of the periodic function $f(x) = x + x^2, -\pi < x \leq \pi$ at $x = \pi$ converges to

(i) $\pi$ (ii) $2\pi$

(iii) $\pi^2$ (iv) $\pi + \pi^2$

(f) If $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \cdots, \lambda_n$ are the eigenvalues of the matrix $A$, then $\lambda_1 - 2$, $\lambda_2 - 2$, $\lambda_3 - 2$, $\lambda_4 - 2, \cdots, \lambda_n - 2$ shall be the eigenvalues of the matrix

(i) $A$ (ii) $A - I$

(iii) $A - 2I$ (iv) $A - 3I$

(g) If $X$ is the eigenvector of matrix $A$, then which is not the eigenvector of $A^{-1}$?

(i) $-X$ (ii) $X^{-1}$

(iii) $X$ (iv) $2X$

(h) If $\lambda$ is an eigenvalue of matrix $A$, then $\frac{1}{\lambda}$ is the eigenvalue of

(i) $A^{-2}$ (ii) $A^{-1}$

(iii) $A^2$ (iv) $A$



(i) The maximum value of $f(x) = x^n(1-x)^n$ for a natural number $n \geq 1$ and $0 \leq x \leq 1$ is

(i) $\frac{1}{4^n}$ (ii) $\frac{1}{3^n}$

(iii) $\frac{1}{2^n}$ (iv) 1

(j) The following series

$$1 + \frac{1}{2^p} + \frac{1}{4^p} + \frac{1}{6^p} + \frac{1}{8^p} + \cdots$$

is convergent for

(i) $p < 1$ (ii) $p \geq 1$

(iii) $p \leq 1$ (iv) $p > 1$

2. (a) Find the evolutes of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Evaluate the integral $\int_0^\infty x^n e^{-k^2x^2} dx$, if $n > -1$. 7+7

3. (a) Find the area of the surface formed by the revolution of $x^2 + 4y^2 = 16$ about its major axis.

*(b)* Evaluate the following integral :

$$\int_0^1 \int_0^{\sqrt{(2x-x^2)}} (x^2 + y^2) dy\, dx$$ 7+7

4\. *(a)* If

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{21} f''(x + \theta h)$$

find the value of $\theta$ as $x \to a$, $f(x)$ being $(x-a)^{5/2}$.

*(b)* Test the convergence of the series

$$\left(\frac{1}{2\cdot 4}\right)^p + \left(\frac{1\cdot 3}{2\cdot 4\cdot 6}\right)^p + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}\right)^p + \cdots$$ 7+7

5\. *(a)* Show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at (0, 0) but partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ both exist and have value 0.

*(b)* Show that the right circular cylinder of given surface (including the ends) and maximum volume is such that its height is equal to the diameter of the base. 7+7





6\. Expand in Fourier series $f(x) = \begin{cases} -x & -4 \le x \le 0 \\ x & 0 \le x \le 4 \end{cases}$ and hence deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \cdots$$ 14

7\. *(a)* Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2i - j - 2k$.

*(b)* Find (i) $\vec{A} \times (\nabla\varphi)$, (ii) $(\nabla \times \vec{A}) \times \vec{B}$, where

$$\vec{A} = x^2zi + yz^3j - 3xyk$$

$$\vec{B} = 3xi + 4zj - xyk \text{ and } \varphi = xy^2z$$ 7+7

8\. Investigate for what values of **a** and **b**, the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \mathbf{a}z = \mathbf{b}$ have *(i)* no solution, *(ii)* a unique solution, *(iii)* an infinite number of solutions.  14

9\. *(a)* Determine the rank and nullity of the following linear transformation :

$$T: V_4 \to V_3$$

defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 - x_2, x_2 + x_3, x_3 - x_4)$$

(b) Determine the eigenvalues and eigenvectors of the following matrix :

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

7+7

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