Code : 211405

B.Tech. 4th Semester Exam., 2014

DISCRETE MATHEMATICAL STRUCTURE AND GRAPH THEORY

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct alternative from the following (any *seven*) : 2×7=14

(a) Consider a simple connected graph G with n vertices and n edges ($n > 2$). Then which of the following statements is True?

(i) G has at least one cycle

(ii) G has no cycles

(iii) The graph obtained by removing any edge from G is not connected

(iv) G has at least one cycle and the graph obtained by removing any edge from G is not connected

(b) The number of distinct simple graphs with up to three nodes is

(i) 9

(ii) 7

(iii) 10

(iv) 15

(c) Consider the graph G, where $V(G) = \{A, B, C, D\}$ and $E(G) = [\{A, B\}, \{B, C\}, \{C, D\}]$. The degree of each of the vertices A, B, C and D respectively in G are

(i) 1, 2, 3, 2

(ii) 1, 3, 2, 2

(iii) 1, 1, 1, 1

(iv) 1, 2, 2, 1

(d) Let f and g be the functions defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$, then the composition of f and g is

(i) $6x + 6$

(ii) $5x + 5$

(iii) $6x + 7$

(iv) $7x + 5$

(e) Among 200 people, 150 either swim or jog, or both. If 85 swim, and 60 swim and jog, how many jog?

(i) 125

(ii) 225

(iii) 85

(iv) 25

(f) A graph in which all nodes are of equal degree is known as —— graph.

(i) complete

(ii) multi

(iii) non-regular

(iv) regular

(g) The minimum number of spanning trees in a connected graph with n nodes is

(i) $n-1$

(ii) $n/2$

(iii) 2

(iv) 1

(h) The negation of 'Today is Friday' is

(i) Today is Saturday

(ii) Today is not Friday

(iii) Today is Thursday

(iv) Today is Sunday

(i) Whether the relation R on the set of all integers is reflexive, symmetric, anti-symmetric or transitive, where $(x, y) \in R$ if and only if $xy \geq 1$?

(i) Anti-symmetric

(ii) Transitive

(iii) Symmetric

(iv) Both symmetric and transitive

(j) If p = 'It is raining' and q = 'She will go to college', then 'It is raining and she will not go to college' will be denoted by

(i) $p \wedge \neg q$

(ii) $p \wedge q$

(iii) $\neg p \wedge q$

(iv) $\neg(p \wedge q)$

2. (a) Define the following terms and give an example for each :

Reflexive, Irreflexive, Anti-symmetric, Transitive, Partition set

(b) If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$ and $A_3 = \{5\}$, define relation R on A by xRy, if x and y are in the same subset A_i, for $1 \leq i \leq 3$. Is R an equivalence relation? 10+4=14

3. *(a)* Let $f, g : z^+ \to z^+$, where $\forall x \in z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$, the maximum of 1 and $x - 1$.

(i) What is the range of f?

(ii) Is f an onto function?

(iii) Is the function one-to-one?

(iv) What is the range of g?

(b) Let $A = \{1\ 2\ 3\ 4\}$. Let R be a relation on A defined by xRy iff x/y and $y = 2x$.

(i) Whether R is a relation of set of ordered pairs?

(ii) Draw digraph of R.

(iii) Determine in-degrees and out-degrees of a digraph. 8+6=14

4. *(a)* Define the following with example :

Group, Subgroup, Homomorphism, Cyclic group, Coset

(b) Determine whether f is one-one or onto for the following cases :

(i) Let $A = B = \{1\ 2\ 3\ 4\}$ and $f = \{(1\ 1)(2\ 3)(3\ 4)(4\ 2)\}$

(ii) Let $A = \{abc\}$, $B = \{1\ 2\ 3\ 4\}$ and $f = \{(a\ 1)(b\ 1)(c\ 4)\}$ 10+4=14

5. *(a)* State and prove De Morgan's laws of set theory.

(b) In a survey of 260 college students, the following data were obtained :

64 had taken a mathematics course

94 had taken a computer science course

58 had taken a business course

28 had taken both a mathematics and a business course

26 had taken both a mathematics and a computer science course

22 had taken both a computer science and a business course

14 had taken all the three types of course.

(i) How many of these students had taken none of the three courses?

(ii) How many had taken only a computer science course? 4+10=14

6. *(a)* Prove that for any non-empty binary tree T, if n_0 is the number of leaves and n_2 be the number of nodes having degree two, then $n_0 = n_2 + 1$.

(b) Derive total number of nodes of a binary tree having depth n. 14

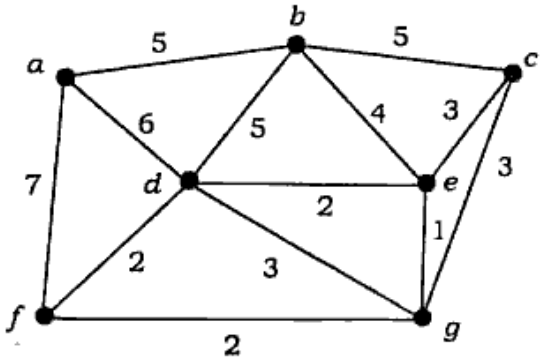
7. *(a)* Define 'group', 'order of a group', and 'Abelian group'.

(b) For $P = \{p_1, p_2, ..., p_5\}$ and $Q = \{q_1, q_2, ..., q_5\}$, explain why $(P, *)$ and $< Q, \Delta >$ are not groups. The operations $*$ and Δ are given in the following table :

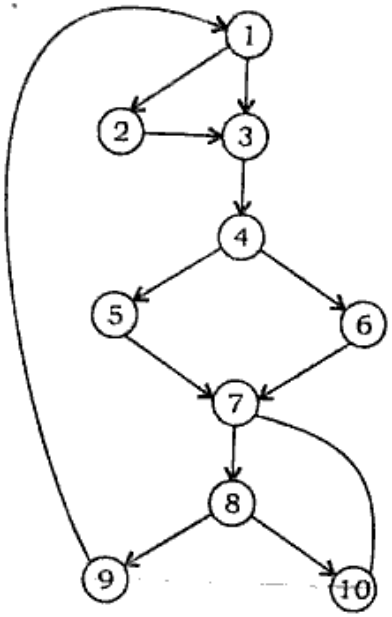
*	p_1	p_2	p_3	p_4	p_5	Δ	q_1	q_2	q_3	q_4	q_5
p_1	p_1	p_2	p_3	p_4	p_5	q_1	q_4	q_1	q_5	q_3	q_2
p_2	p_2	p_1	p_4	p_5	p_3	q_2	q_3	q_5	q_2	q_1	q_4
p_3	p_3	p_5	p_1	p_2	p_4	q_3	q_1	q_2	q_3	q_4	q_5
p_4	p_4	p_3	p_5	p_1	p_2	q_4	q_2	q_4	q_1	q_5	q_3
p_5	p_5	p_4	p_2	p_3	p_4	q_5	q_5	q_3	q_4	q_2	q_1

6+8=14

8. Explain Dijkstra's algorithm and apply it to the weighted graph $G = (V, E)$ shown in Figure below and determine the shortest distance from vertex a to each of the other vertices in the graph. 6+8=14



9. Consider the graph given below :



(a) Find the adjacency list and BFS traversal of the above graph.

(b) Prove that the maximum number of edges possible in a simple graph of n nodes is $n(n-1)/2$. 8+6=14

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