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Code : 031815

B.Tech 8th Semester Exam., 2021

MODERN CONTROL THEORY

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are* **NINE** *questions in this paper.*

(iii) *Attempt* **FIVE** *questions in all.*

(iv) *Question No.* **1** *is compulsory.*

1. Answer any *seven* of the following : $2 \times 7 = 14$

(a) Define controllability and observability.

(b) Define similarity transformation and its significance.

(c) Define controllable canonical and observable canonical forms.

(d) Define deadzone non-linearity.

(e) Explain the requirement of pole-placement design.

(f) Define state variables and transfer function.

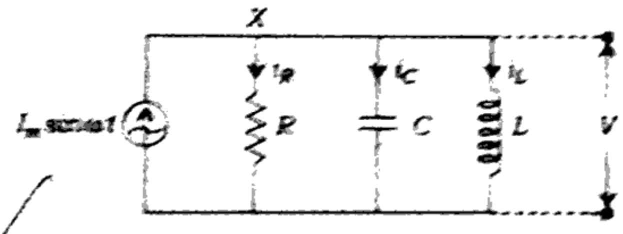
(g) Define global Lyapunov stability.

(h) Give the solution of non-homogeneous state-space equation.

(i) Explain reduced order observer design.

(j) Define nilpotent matrix. Where may it be used?

2. (a) Obtain the state model of the parallel *RLC* network as shown below :



(b) Consider the state-space model of an LTI system with matrices

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}^T$$

and find its transfer function. 7+7=14

3. (a) Consider the state-space model of an LTI system with matrices

$$A = \begin{bmatrix} -2 \cdot 5 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 3 \cdot 5 \\ 0 \end{bmatrix}$$

Find the state transition matrix and comment on the controllability.

(b) Consider the LTI system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -0 \cdot 7 & -0 \cdot 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Find the non-homogeneous solution if $x_1(0) = 3 \cdot 5$, $x_2(0) = 0$ and u is a unit step function. 7+7=14

4. (a) A regulator system has a plant

$$\frac{Y(s)}{U(s)} = \frac{24}{(s+1 \cdot 5)(s+4 \cdot 5)(s+3 \cdot 5)}$$

Define state variable as $x_1 = 2y$, $x_2 = 3x_1$, $x_3 = 5x_2$. By use of the state feedback control $u = -Kx$, it is desired to place the closed-loop poles at $s = -4 \pm \sqrt{3}j$ and $s = -12$. Determine the necessary state feedback gain matrix K.

(b) Consider the system given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-2 & 0 & 1\\ 1 & -2\cdot5 & 0\\ 0 & 0 & -4\cdot3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\cdot6\end{bmatrix}u$$

$$y=[2\cdot1 \quad 1 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Transform the system equations into
(i) controllable canonical form and
(ii) observable canonical form. 7+7=14

5. *(a)* Consider the system defined by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1\cdot78 & 0\cdot55 & -5\cdot5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\cdot54\end{bmatrix}u$$

$$y=[1 \quad 0 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Given the set of desired poles for the observer to be $s=-4\cdot2\pm1\cdot3\sqrt{3}j$ and $s=-11$. Design a full-order observer.

(b) Consider the system given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-4 & -5 & -2\\ 0 & -1 & 3\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}7\\ 0\\ 1\end{bmatrix}u$$

$$y=[4 \quad 1 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Is the system completely state controllable and completely observable? Explain how to obtain a transformation matrix in similarity transformation. 7+7=14

6. *(a)* If the state equation of a system is described by

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

find eigenvalues, eigenvectors and state transition matrix. Also determine the solution of the system with input $u(t)=1$ for $t\geq0$ and initial vector $x_1(0)=1$, $x_2(0)=-1$.

(b) Consider the LTI system

$$\dot{x}=\begin{bmatrix}0 & 1\\ -2 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 2\end{bmatrix}u$$

Find the non-homogeneous solution if $x_1(0)=1$, $x_2(0)=-2$ and u is a unit step function. 7+7=14

7. *(a)* Consider the state-space model of an LTI system with matrices

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Find the state transition matrix and comment on the controllability.

(b) Consider

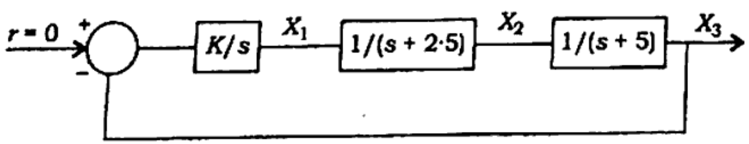
$$\dot{x} = \begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \; ; \; y = \begin{bmatrix} 1 & 0 \cdot 5 \\ 0 & 2 \cdot 5 \end{bmatrix} x$$

Find the optimal control law that minimizes

$$J = \frac{1}{2}\int_0^{\infty} (y_1^2 + 2y_2^2 + 2 \cdot 5u^2)\, dt$$

7+7=14

8. *(a)* Using Lyapunov's direct method, find the range of K to guarantee stability of the system shown in the figure below. Use the Lyapunov's direct method and Routh criterion and compare the results :



(Continued)

(b) Explain singular points. Find out the singular points for the following systems :

(i) $y'' + 5y' - 20 = 0$

(ii) $y'' + 3 \cdot 5y' + 2 \cdot 5y = 0$

Show the trajectories of the singular points. 7+7=14

9. *(a)* For a sinusoidal input $x = X \sin \omega t$, find the output waveforms for the following non-linearity :

$$N(X) = \begin{cases} 0 & ; \; X < \Delta \\ \dfrac{2M}{\pi X}\sqrt{1 - \left(\dfrac{\Delta}{X}\right)^2} & ; \; X \geq \Delta \end{cases}$$

By Fourier series analysis of the output waveforms, derive the describing function.

(b) Discuss friction controlled backlash and describing functions. 8+6=14

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