Code : 103102

B.Tech 1st Semester Exam., 2018 (New)

MATHEMATICS—I

(Calculus and Differential Equations)

Time : 3 hours

Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct answer of the following (any *seven*) : 2×7=14

(a) The maximum value of $\sin x+\cos x$ is

(i) 1

(ii) 2

(iii) $\sqrt{2}$

(iv) 0

(b) The value of the integral

$$\int_C \{yzdx+(xz+1)\,dy+xydz\}$$

where C is any path from (1, 0, 0) to (2, 1, 4) is

(i) 6

(ii) 7

(iii) 8

(iv) 9

(c) The series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\cdots$$

is

(i) convergent

(ii) divergent

(iii) absolutely convergent

(iv) None of the above

(d) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to

(i) $\tan 2u$

(ii) $\cos 2u$

(iii) $\sin 2u$

(iv) $\cot 2u$

(*Continued*)

(e) If $A(x, y, z) = x^2 z i - 2y^3 z^2 j + xy^2 z k$, then $(\nabla \cdot A)$ at the point $(1, -1, 1)$ is

(i) 3

(ii) −3

(iii) 4

(iv) −4

(f) The value of $\iint dx dy$ over the region $9x^2 + 4y^2 \leq 4$ is

(i) π

(ii) 2π

(iii) $\frac{\pi}{3}$

(iv) $\frac{2\pi}{3}$

(g) If P_n is the Legendre polynomial of first kind, then the value of $\int_{-1}^{1} x P_n P_n' \, dx$ is

(i) $\frac{2}{(2n+1)}$

(ii) $\frac{2n}{(2n+1)}$

(iii) $\frac{2}{(2n+3)}$

(iv) $\frac{2n}{(2n+3)}$

(h) If J_n is the Bessel's function of first kind, then the value of $J_{-\frac{1}{2}}$ is

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\sqrt{\frac{2}{\pi x}} \sin x$

(iv) $\sqrt{\frac{2}{\pi x}} \cos x$

(i) For the differential equation $t(t-2)^2 y'' + ty' + y = 0$, $t = 0$, is

(i) an ordinary point

(ii) a branch point

(iii) an irregular point

(iv) a regular singular point

(j) The solution of the equation $xp^3q^2 + yp^2q^3 + (p^3 + q^3) - zp^2q^2 = 0$ is $z =$

(i) $ax + by + (ab^{-2} + ba^{-2})$

(ii) $ax - by + (ab^{-2} - ba^{-2})$

(iii) $-ax + by + (-ab^{-2} + ba^{-2})$

(iv) $ax + by - (ab^{-2} + ba^{-2})$

2. *(a)* Find the evolutes of the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

where a is the constant.

(b) Show that

$$\Gamma(1/4)\cdot\Gamma(3/4) = 2\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta = 4\int_0^{\infty}\frac{x^2}{1+x^4}\,dx = \pi\sqrt{2}$$

7+7=14

3. *(a)* Expand $f(x, y) = x^2y + 3y - 2$ in the powers of $(x+2)$ and $(y-1)$ by Taylor's theorem.

(b) Find the maxima and minima, if any, of

$$f(x) = \frac{x^4}{(x-1)(x-3)^3}$$

7+7=14

4. Explain the Dirichlet conditions. Find out the Fourier series for the periodic function defined by

$$f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ x^2, & 0 \le x < \pi \end{cases}$$

and hence find the sum of the series—

(a) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots$;

(b) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \cdots$.

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(Turn Over)

5. *(a)* If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u$$

(b) Find $\nabla\cdot\nabla\varphi$, where $\varphi = xy^2z^4$.

7+7=14

6. *(a)* Evaluate the integral by changing the order of integration

$$\int_0^{\infty}\int_0^{x} x e^{-\frac{x^2}{y}}\,dy\,dx$$

(b) Solve the differential equation

$$(x^2 + y^2 + x)\,dx - (2x^2 + 2y^2 - y)\,dy = 0$$

7+7=14

7. Verify the Stokes' theorem for

$$A = (y - z + 2)i + (yz + 4)j - xzk$$

where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$ and $z = 2$ above the xy-plane.

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8. **Solve in series, using Frobenius method, the equation** $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$.

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(Continued)

9. (a) State and prove orthogonal properties of Legendre polynomials.

(b) Find the complete integral of the equation $(p^2 + q^2)y = zq$. 7+7=14

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