Code : EC–102 (104305)

B.Tech 3rd Semester Special Exam., 2020

SIGNALS AND SYSTEMS

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

(iv) Question No. 1 is compulsory.

1. Answer the following as directed (any seven) : 2×7=14

(a) If the Z-transform of $x(n)$ is $X(z)$, then show that

$$Z[x_1(n)^* x_2(n)] = X_1(z) X_2(z)$$

(b) If the impulse response for a system is given by $h(n) = a^n u(n)$, then what is the condition for the system to be BIBO stable?

(c) A voltage having the Laplace transform

$$\frac{4s^2 + 3s + 2}{7s^2 + 6s + 5}$$

is applied across a 2H inductor. What is the current in inductor at $t = \infty$, assuming zero initial condition?

(d) Differentiate between Kronecker delta function and Dirac delta function.

(e) The step response of an LTI system when the impulse response $h(n)$ is unit step $u(n)$ is _____.

(Fill in the blank)

(f) Find the Laplace transform

$$f(t) = e^{3t} \cos(2t) u(t)$$

where symbols have their usual meanings.

(g) An LTI system is described as

$$0 \cdot 5 \frac{d^2 y(t)}{dt^2} + 2 \cdot 5 \frac{dy(t)}{dt} + 2y(t) = \delta(t)$$

Find the final value of the output response where $y(t)$ is output and $x(t)$ is input.

(h) The period of a sequence

$$x(n) = \cos\left(\frac{2\pi n}{3}\right)$$

is ____.

(Fill in the blank)

(i) The final value of step response of a causal LTI system with

$$H(s) = \frac{s+1}{s+4}$$

is

(i) 0·5

(ii) 0·25

(iii) 1

(iv) ∞

(Choose the correct option)

(j) Consider two functions $f(t) = h(t)h(3-t)$ and $g(t) = h(t) - h(t-3)$. Are these two functions identical? Show that

$$L[f(t)] = L[g(t)]$$

where L is the Laplace operator.

2. *(a)* Let a system is described by the differential equation as $\ddot{y} + 3\dot{y} + 2y = e^{-t}$; with initial condition $y(0) = \dot{y}(0) = 0$. Compute the solution of the equation. 5

(b) Let $f(t)$ is a periodic function with periodicity T for $t \geq 0$, then show that

$$L[f(t)] = \frac{L[f_T(t)]}{1 - e^{-sT}} \quad s > 0$$

4

(c) Find the Laplace transform of Fig. 1 : 5

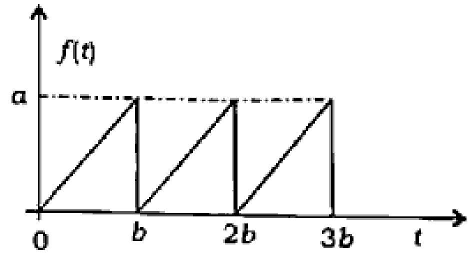


Fig. 1

3. *(a)* State why ROC does not include any pole. Find the Z-transform of

$$x(n) = \begin{cases} (0\cdot5)^n u(n), & n > 0 \\ (0\cdot25)^{-n}, & n < 0 \end{cases}$$

1+5

(b) Find the inverse Z-transform of

$$X(z) = \frac{1 - \frac{1}{4}z^{-1}}{1 - \frac{1}{9}z^{-1}}$$

where ROC : $|z| > \frac{1}{3}$. 5

(c) Show that

$$Z[nx(n)] = -z\frac{dX(z)}{dz}$$

where $X(z) \xleftrightarrow{z} x[n]$. 3

4. (a) Briefly explain the causality of a system. 2

(b) Find whether the signal

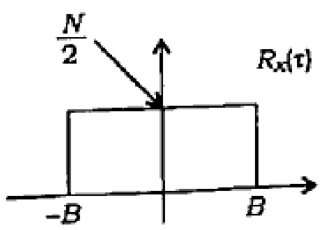
$$x[n] = \sin\left(\frac{3\pi}{4}n\right) + \sin\left(\frac{\pi}{3}n\right)$$

is periodic or aperiodic. If periodic, then what is the periodicity of $x[n]$? 4

(c) Write down the Dirichlet condition. 2

(d) Find the Fourier transform of $x(t) = e^{-|t|}u(t)$, and hence draw the magnitude and phase spectrums. 6

5. (a) Compute the Fourier transform of signal shown in Fig. 2 : 5



$$R_x(\tau) = \begin{cases} \frac{N}{2}, & -B \le \tau \le B \\ 0, & \text{elsewhere} \end{cases}$$

Fig. 2

(b) Find the convolution of the following discrete sequences : 4

$$x(n) = \frac{1}{3}u(n) \text{ and } h(n) = \frac{1}{5}u(n)$$

(c) State why the realization of an ideal low-pass filter is not possible, with proper justification. 5

6. (a) A system is defined as $y(n) = x(n^2)$. Check whether the system is linear or non-linear, time-varying or time-invariant, causal or non-causal, and memoryless or memory type. 5

(b) State Parseval's theorem. 3

(c) Sketch the signal $x(t) = -2u(t-1)$. 4

(d) Compute the Nyquist sampling rate for the signal

$$g(t) = 10\cos(50\pi)\cos^2(150\pi t)$$ 2

7. (a) Show that

$$u(n) = \sum_{n=-\infty}^{\infty} \delta(n)$$

where symbols have their usual meanings. 2

(b) What is unit doublet? Prove that

$$\int_{-\infty}^{\infty} \delta^k(t)x(t)dt = (-1)^k \frac{d^k x(0)}{dt^k}$$

where $\left| \frac{d^k}{dt^k} x(t) \right|$ is k-th derivative of function $x(t)$, and $\delta(t)$ is Dirac delta function. 1+3=4

(c) A system is described by its input-output relationship as

$$y[n] = \sum_{k=-\infty}^{0} x[n-k]$$

Is the system memoryless, stable, causal, time-invariant and linear? 5

(d) Find the fundamental period of signal

$$x[n] = e^{j7 \cdot 351 \pi n}$$ 3

8. *(a)* Let $x[n]$ be an arbitrary function with even and odd part as $x_e[n]$, $x_0[n]$, respectively. Show that

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_0^2[n]$$ 4

(b) Perform the convolution operation between

$$x[n] = \{0, 0, 0, 0, \underset{\uparrow}{2}, -3, 1, 0, 0\}$$

and $$h[n] = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 0, 0, 0\}$$

using graphical method. 6

(c) Calculate the Fourier transform of

$$x[n] = u[n]$$ 4

9. Write short notes on any *four* of the following : 3½×4=14

(a) Nyquist sampling theorem

(b) Evolution of Fourier series coefficient

(c) Initial and final value theorems of Laplace transform

(d) BIBO stability

(e) Zero-order hold circuit

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