Code : 103202

B.Tech 2nd Semester Exam., 2019

MATHEMATICS—II

(Linear Algebra, Transform Calculus and Numerical Method)

(New Course)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) If A is a 3-rowed square matrix such that $|A|=2$, then $|\text{adj}\{\text{adj}(\text{adj}\,A^2)\}|$ is equal to

(i) 2^4

(ii) 2^8

(iii) 2^{16}

(iv) None of the above

(b) If 3, −2 are the eigenvalues of a non-singular matrix A and $|A|=4$, then eigenvalues of adj A are

(i) $\frac{3}{4}, -\frac{1}{2}$

(ii) $\frac{4}{3}, -2$

(iii) 12, −8

(iv) None of the above

(c) Let A be a skew-symmetric matrix of order n, then

(i) $|A|=0$, if n is even

(ii) $|A|=0$, if n is odd

(iii) $|A|=0$ for all $n \in N$

(iv) $|A|\neq 0$, always

(d) If A is non-zero column matrix of the type $n\times 1$ and B is non-zero row matrix of the type $1\times n$, then $\rho(AB)$ is

(i) 0

(ii) 1

(iii) n

(iv) None of the above

(e) In regula-falsi method, the first approximation is given by

(i) $x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

(ii) $x_1 = \dfrac{bf(b) - af(a)}{f(b) - f(a)}$

(iii) $x_1 = \dfrac{bf(a) + af(b)}{f(a) - f(b)}$

(iv) $x_1 = \dfrac{af(a) - bf(b)}{f(a) - f(b)}$

(f) While evaluating the definite integral by trapezoidal rule, the accuracy can be increased by taking

(i) large number of sub-intervals

(ii) even number of sub-intervals

(iii) $h = 4$

(iv) a multiple of 3

(g) Various types of Runge-Kutta methods are classified according to their

(i) degree

(ii) order

(iii) rank

(iv) Both (i) and (ii)

(h) The value of $L\left\{\dfrac{\cos 10t}{t}\right\}$ is

(i) 0

(ii) 1

(iii) 2

(iv) Does not exist

(i) Laplace transform of unit step function is

(i) $\dfrac{e^{-as}}{s}$ (ii) $\dfrac{e^{as}}{s}$

(iii) $\dfrac{e^{-as}}{s+1}$ (iv) $\dfrac{e^{as}}{s+1}$

(j) Which function has Laplace transform even it is not piecewise continuous in the range?

(i) $\dfrac{1}{\sqrt{t}}$

(ii) $\dfrac{1}{\sqrt{t^2}}$

(iii) $\dfrac{1}{\sqrt{t^3}}$

(iv) All of the above

2. *(a)* Investigate for what value of λ and μ do the system of equations $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$ have *(i)* no solution, *(ii)* unique solution and *(iii)* infinite number of solution. 7

(b) Find the eigenvalues and eigenvectors of the matrix

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$ 7

3. *(a)* Verify Cayley-Hamilton theorem for the matrix

$$A=\begin{bmatrix}1 & 2 & 0\\ -1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$$

Also obtain *(i)* A^{-1}, *(ii)* eigenvalues of A and A^2, and *(iii)* spectral radius of A. 8

(b) Diagonalize the matrix

$$A=\begin{bmatrix}2 & 0 & 4\\ 0 & 6 & 0\\ 4 & 0 & 2\end{bmatrix}$$

by means of an orthogonal transformation. 6

4. *(a)* Find a real root of the equation $x\log_{10}x=1{\cdot}2$ by using regula-falsi method correct to four significant digits. 7

(b) Show that the following two sequences both have convergence of the second order with the same limit $\sqrt{a}$: 7

$$x_{n+1}=\frac{1}{2}x_n\left(1+\frac{1}{x_n^2}\right) \quad \text{and} \quad x_{n+1}=\frac{1}{2}x_n\left(3-\frac{x_n^2}{a}\right)$$

5. *(a)* Derive Newton's forward interpolation formula. 7

(b) Find the value of cos51°43′ by Gauss's backward interpolation formula. Given that

x	50°	51°	52°	53°	54°
cos x	0·6428	0·6293	0·6157	0·6018	0·5878

7

6. *(a)* Solve the differential equation $\frac{dy}{dx}=y-x^2$ by Milne's method and compute y at $x=0{\cdot}80$. Given that

x	0·0	0·2	0·4	0·6
y	1	1·12186	1·46820	1·73790

8

(b) Using Adams-Moulton-Bashforth method, find $y(1\cdot4)$. Given

$$\frac{dy}{dx} = x^2(1+y),\ y(1)=1,\ y(1\cdot1)=1\cdot233,$$
$$y(1\cdot2)=1\cdot548,\ y(1\cdot3)=1\cdot979 \quad 6$$

7. (a) Solve $u_{xx} = u_t$ in $0 < x < 2$, $t > 0$, $u(0, t)=u(2, t)=0$, $t > 0$ and $u(x, 0)=\sin(\pi, x/2)$, $0 \le x \le 2$ using $\Delta x = 0\cdot5$, $\Delta t = 0\cdot25$ for one time step by Crank-Nicolson implicit finite difference method. 10

(b) Write an implicit method for solving the one-dimensional wave equation $u_{tt} = c^2 u_{xx}$, $0 \le x \le l$, $t > 0$. 4

8. (a) Evaluate

$$\int_0^\infty \left\{\cos t \cdot \delta\left(t - \frac{\pi}{4}\right)\right\} dt$$

by using Laplace transform. 7

(b) Find the Fourier transform of the function $f(t) = e^{-a|t|}$, $-\infty < t < \infty$, $a > 0$. 7

9. (a) Find the inverse Laplace transform of

$$\tan^{-1}\left(\frac{2}{s^2}\right) \quad 6$$

(b) Solve the given partial differential equation by Laplace transform : 8

$$x\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = xt, \text{ if } u(x, 0) = 0,\ u(0, t) = t$$

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