Code : 303202

BCA 2nd Semester Exam., 2018

MATHEMATICS (Numerical Techniques)

Time : 3 hours *Full Marks* : 60

Instructions :

(i) All questions carry equal marks.

*(ii) There are **SEVEN** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question Nos. **1** & **2** are compulsory.*

1. Choose the correct answer of the following (any *six*) :

(a) The number of significant digits in the number 204·020050 is

(i) 5

(ii) 6

(iii) 8

(iv) 9

(Turn Over)

(b) The convergence of which of the following methods is sensitive to starting value?

(i) False position

(ii) Gauss-Seidel method

(iii) Newton-Raphson method

(iv) All of the above

(c) Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If iterations are started from −1, then iterations will

(i) converge to −1

(ii) converge to $\sqrt{2}$

(iii) converge to $-\sqrt{2}$

(iv) No convergence

(d) In the Gauss elimination method for solving a system of linear algebraic equations, triangularization leads to

(i) diagonal matrix

(ii) lower triangular matrix

(iii) upper triangular matrix

(iv) singular matrix

(Continued)

(e) In which of the following methods, we approximate the curve of solution by the tangent in each interval?

(i) Picard's method

(ii) Euler's method

(iii) Newton's method

(iv) Runge-Kutta method

(f) Match the following :

A.	Newton-Raphson	1.	Integration
B.	Runge-Kutta	2.	Root finding
C.	Gauss-Seidel	3.	Ordinary differential equations
D.	Simpson's rule	4.	Solution of system of linear equations

The correct matching is

(i) A–2, B–3, C–4, D–1

(ii) A–3, B–2, C–1, D–4

(iii) A–1, B–4, C–2, D–3

(iv) A–4, B–1, C–2, D–3

(g) If $\Delta f(x) = f(x+h) - f(x)$, then a constant k, Δk equals

(i) 1

(ii) 0

(iii) $f(k) - f(0)$

(iv) $f(x+k) - f(x)$

(v) None of the above

(Turn Over)

(h) A root of the equation $x^3 - x - 11 = 0$ correc to four decimals using bisection method is

(i) 2·3737

(ii) 2·3838

(iii) 2·3736

(iv) None of the above

(i) The order of errors in the Simpson's rule fo numerical integration with a step size h is

(i) h

(ii) h^2

(iii) h^3

(iv) h^4

(j) In which of the following methods prop choice of initial value is very important?

(i) Bisection method

(ii) False position

(iii) Newton-Raphson method

(iv) None of the above

(Continued

2. Answer any *three* of the following :

(a) Use quadratic convergent method to find the first approximation $x^{(1)}$ of $\sqrt[3]{28}$ if $x^{(0)} = 3$.

(b) Show that the Newton method for finding reciprocals by solving $(1/x) - c = 0$ results in the iteration, $x_{n+1} = x_n(2 - cx_n)$, $n \geq 0$.

(c) Let $g(x)$ be a continuous function on $[a, b]$, and suppose that g satisfies the property $a \leq x \leq b$ implies that $a \leq g(x) \leq b$. Then the equation $x = g(x)$ has at least one solution in $[a, b]$.

(d) The function $f(x)$ has the exact values shown in the table below :

x	1	3	5
$f(x)$	4	−2	10

Using Newton's forward difference interpolation method, estimate the value of $f(6)$.

(e) Find the absolute and the relative error when $x = 3 \cdot 162$ is used as an approximation to $x = \sqrt{10}$.

3. Show that equation $xe^x - 1$ has a root in $[1/2, 1]$ and find the approximation to this root within 10^{-1}, using bisection method.

(Turn Over)

4. Evaluate the integral $\int_0^{\pi/4} \sin 4x\,dx$, using trapezoidal rule with $n = 4$. Estimate the error bound and compute with exact error.

5. Given the table of values :

x	1·0	1·05	1·08	1·1
$f(x)$	2·72	3·29	3·66	3·90

Construct the best quadratic Lagrange interpolation polynomial to approximate the equation, $f(x) = 3xe^x - 2e^x$ at $x = 1 \cdot 04$.

6. Consider the linear systems :

$$\begin{aligned} 0 \cdot 5x_1 + 1 \cdot 1x_2 + 3 \cdot 1x_3 &= 6 \cdot 0 \\ 2 \cdot 0x_1 + 4 \cdot 5x_2 + 0 \cdot 4x_3 &= 0 \cdot 02 \\ 5 \cdot 0x_1 + 1 \cdot 0x_2 + 6 \cdot 5x_3 &= 1 \cdot 0 \end{aligned}$$

Solve the systems using Gauss elimination method.

7. Use Runge-Kutta of order 2 with $h = 0 \cdot 1$ to find the approximate value of $y(0 \cdot 3)$ of the initial value problem, $y' = y(2 - y)$; $y(0) = 0 \cdot 1$.

* * *