Code : 211405

B.Tech 4th Semester Exam., 2018

DISCRETE MATHEMATICAL STRUCTURE AND GRAPH THEORY

Time : 3 hours

Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) For any three sets A, B and C, which of the following statements is wrong?

(i) $A \cup (B \cup C) = (A \cup B) \cap C$

(ii) $A \cup (B \cup C) = A \cup (B \cup C)$

(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) None of the above

(b) Let A and B be two non-empty sets. Then the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$ is called

(i) product set

(ii) poset

(iii) binary set

(iv) None of the above

(c) If $(a, a) \in R$ or equivalently $a R a, \forall a \in A$, then a relation R on a set A is called

(i) equivalent

(ii) reflexive

(iii) symmetric

(iv) anti-symmetric

(d) Let A and B be finite sets with $|A| = n$ and $|B| = m$. How many functions are possible from A to B with A as the domain?

(i) n

(ii) m^m

(iii) m

(iv) m^n

(e) For the functions f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1 \forall x \in R$, the value of $(g \circ f)(x)$ is

(i) $x^2 + 1$

(ii) $x^3 + 1$

(iii) $x^6 + 1$

(iv) $x^5 + 1$

(f) A group G is said to be Abelian (or commutative) if for every

(i) $a, b \in G$

(ii) $a \cdot b = b \cdot a$

(iii) Both *(i)* and *(ii)*

(iv) None of the above

(g) If $f : G \to G'$ is a homomorphism, then which of the following it true?

(i) $f(e) = e$

(ii) $f(e) = e'$

(iii) $f(e) = 1$

(iv) $f(e) >= \Phi$

(h) For which of the following does there exist a tree satisfying the specified constraints?

(i) A full binary tree with 31 leaves, each leaf of height 5

(ii) A rooted tree of height 3 where every vertex has atmost 3 children and there are 41 total vertices

(iii) a full binary tree with 11 vertices and height 6

(iv) A binary tree with 2 leaves and height 100

(i) For which of the following does there exist a graph $G = (V, E, \varphi)$ satisfying the specified conditions?

(i) A tree with 9 vertices and the sum of the degrees of all the vertices is 18

(ii) A graph with 5 components, 12 vertices and 7 edges

(iii) A graph with 5 components, 30 vertices and 24 edges

(iv) A graph with 9 vertices, 9 edges and no cycles

(v) A connected graph with 12 edges, 5 vertices and fewer than 8 cycles

(j) The number of simple digraphs with $|V| = 3$ is

(i) 2^9

(ii) 2^8

(iii) 2^7

(iv) 2^6

(v) 2^5

2. *(a)* Let $f(x) = ax^2 + b$ and $g(x) = cx^2 + d$, where a, b, c and d are constants. Determine for which constants a, b, c and d the following equation holds : 6

$$f \circ g = g \circ f$$

(b) Show that the relation $(x, y) R (a, b)$ such that

$$x^2 + y^2 = a^2 + b^2$$

is an equivalence relation on the plane and describe the equivalence classes. 8

3. *(a)* Let $(G, *)$ be a group, where $*$ is usual multiplication operation on G. Then show that for any $x, y \in G$, following equations hold : 7

(i) $(x^{-1})^{-1} = x$

(ii) $(xy)^{-1} = y^{-1} x^{-1}$

(b) Construct the truth table for

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

Also show that above statement is a tautology by developing a series of logical equivalences. 7

4. *(a)* If $A = \{1, 2, 4\}$, $B = \{2, 5, 7\}$ and $C = \{1, 3, 7\}$, find $(A \times B)$ $(A \times C)$. 6

(b) List the ordered pairs in the relation R from $A = \{1, 2, 3, 4\}$ to $B = \{2, 3, 4, 5\}$, where $(a, b) \in R$, if and only if—

(i) $a = b$;

(ii) $a + b = 5$. 8

5. *(a)* Show that the set of integers with the composition 0 and $*$ defined by $a \circ b = a + b + 1$ and $a * b = ab + a + b$ is a ring. 7

(b) State and prove Lagrange's theorem. 7

6. *(a)* Define a relation R on the set Z of all integers as follows :

$m R n \Leftrightarrow m + n$ is even for all $m, n \in Z$. Is R a partial order relation? Prove or give a counter example. 7

(b) Show that the group *(i)* $\{\{1, 2, 3, 4\}, X_5\}$, *(ii)* $\{\{1, 2, 3, 4, 5, 6\}, X_7\}$ is cyclic. 7

7. *(a)* Let $A = \{0, 1, 2, 3\}$, $R = \{(x, y) : x + y = 3\}$, $S = \{(x, y) : 3 / (x + y)\}$, $T = \{(x, y) : \max(x, y) = 3\}$

Compute *(i)* $R \circ T$, *(ii)* $T \circ R$ and *(iii)* $S \circ S$. 7

(b) In a group of 70 cars tested by a garage in Delhi, 15 had faulty tyres, 20 had faulty breaks and 18 exceeded the allowable emission limits. Also, 5 cars had faulty tyres and brakes, 6 failed on tyres and emission, 10 failed on brakes and emissions, and 4 cars were unsatisfactory in all three respects. How many cars had no faults in these three checks? Draw an appropriate Venn diagram. 7

8. *(a)* Define the vertex connectivity and edge connectivity of a graph. Prove that for a G with n vertices and e edges, vertex connectivity $\leq$ edge connectivity $\leq 2\,e/n$. 7

(b) Define the adjacency matrix of a graph. Find the rank of the regular graph with n vertices and with degree $p(< n)$ of every vertex. 7

9. Write short notes on any *three* of the following : 14

(a) Multigraphs

(b) Planar graphs

(c) Cosets

(d) Ring polynomials

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