Code : 100312

B.Tech 3rd Semester Exam., 2021

(New Course)

MATHEMATICS—III

(PDE, Probability and Statistics)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

(v) Statistical data is given at the end.

(vi) Non-programmable scientific calculator is allowed.

1. Choose the correct answer (any *seven*) : 2×7=14

(a) If $z = f\left(\frac{xy}{z}\right)$ satisfies the partial differential equation $x^{\alpha}p - yq = \beta$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, then the value of (α, β) is

(i) (1, 0)

(ii) (0, 1)

(iii) (0, 0)

(iv) (1, 1)

(b) The particular integral of the PDE

$$xy\frac{\partial^2 z}{\partial x \partial y} = 1$$

is $\log(ax)\log(by)$. Then the value of (a, b) is

(i) (1, 1)

(ii) (2, 1)

(iii) (2, 3)

(iv) (1, 2)

(c) The solution of the PDE

$$y^2 zp + x^2 zq = y^2 x$$

is $x^{\alpha} - y^{\beta} = f(x^2 - z^2)$. Then the value of (α, β) is

(i) (2, 3)

(ii) (3, 3)

(iii) (3, 1)

(iv) (1, 1)

(d) Let A and B are two possible outcomes of an experiment and suppose $P(A) = 0 \cdot 3$, $P(B) = \frac{K}{7}$ and $P(A \cup B) = 0 \cdot 6$.

If A and B are independent events, then the value of K is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(e) The first three moments of a distribution about 5 are 2, 7, 15 respectively. Then the mean and variance of the distribution are

(i) (5, 15)

(ii) (7, 15)

(iii) (7, 52)

(iv) (7, 3)

(f) If two dice are thrown and the probability that the sum is neither 8 nor 12 is $\frac{5}{k}$, then the value of k is

(i) 3

(ii) 4

(iii) 6

(iv) 12

(g) A random variable X has mean 2 and variance 3. If the upper bound for $P(|X-2|>6)$ by using Chebyshev's inequality is $\frac{1}{K}$, then the value of K is

(i) 9

(ii) 12

(iii) 15

(iv) 18

(h) If the mean of exponential distribution

$$f(x)=\begin{cases} ke^{-kx}, & x>0 \\ 0, & x\le 0 \end{cases}$$

is $\frac{1}{6}$, then the value of k is

(i) 4

(ii) 5

(iii) 6

(iv) 7

(i) Let X and Y have joint p.d.f.

$$f(x,y)=\begin{cases} cxy; & 0<x<2,\ 0<y<1 \\ 0; & \text{elsewhere} \end{cases}$$

and $E(xy)=\frac{p}{q}$. Then the value of (p, q) is

(i) (4, 9)

(ii) (4, 4)

(iii) (8, 9)

(iv) (9, 4)

(j) Two random variables have the least square regression lines with equations $2x+y=5$ and $3x+2y=8$. Then the correlation coefficient between two variables x and y is

(i) $\frac{\sqrt{3}}{2}$

(ii) $-\frac{\sqrt{3}}{2}$

(iii) $\frac{1}{2}$

(iv) $-\frac{1}{2}$

2. (a) Obtain the partial differential equation by eliminating the arbitrary function

$$u = f(x^2+y^2+z^2,\ z^2-2xy)$$

(b) Solve

$$z^2(p^2+q^2) = x^2+y^2$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 7+7

3. (a) Solve the partial differential equation $(2D^2 - DD' - 3D'^2)z = 5e^{x-y}$.

(b) Prove that

$$(1-x^2)P_n'(x) = n[P_{n-1}(x) - xP_n(x)]$$

where $P_n(x)$ is the Legendre's polynomial of the first kind. 7+7

4. Two ends A and B of a rod of length 20 cm have the temperatures 30 °C and 80 °C, respectively, until the steady-state conditions prevail. Then the temperatures at the ends A and B are changed to 40 °C and 60 °C, respectively. Find the temperature distribution of the rod at any time t. 14

5. (a) Let A, B and C be three independent events. Then show that B and $A \cap \overline{C}$ are also independent.

(b) A coin is tossed. If it turns up H, two balls will be drawn from urn A, otherwise 2 balls will be drawn from urn B. Urn A contains 3 red and 5 blue balls, urn B contains 7 red and 5 blue balls. What is the probability that urn A is used? It is given that both balls are blue.

(Find in both cases, when balls were chosen with replacement and without replacement.)

 7+7

6. *(a)* If X and Y are independent Poisson variates such that $P(X=1)=P(X=2)$ and $P(Y=2)=P(Y=3)$. Find the variance of $X-2Y$.

   (b) Show that the recurrence relation between the moments of binomial distribution is given by

$$\mu_{r+1}=\left(nr\mu_{r-1}+\frac{d\mu_r}{dp}\right)pq$$

   where μ_r is the rth order moment about the mean. 7+7

7. Given the joint probability density function of X and Y as

$$f(x,y)=\begin{cases}kxy; & 0<x<4,\ 0<y<3\\ 0; & \text{elsewhere}\end{cases}$$

   (a) Compute the probability density function of $X+2Y$.

   (b) Compute the following :

   (i) $P(1<y\le 2)$

   (ii) $P(1<x+2y\le 3)$

   (iii) $P(x>1/y\le 2)$ 8+6

8. The table gives the stopping distance y (feet) of an automobile travelling at speed x (miles per hour) as the instant danger is sighted :

x	30	45	60	75	90	105
y	16	27	40	60	87	115

   (a) Fit a least-square parabola of the form $y=a+bx+cx^2$ to the data.

   (b) Estimate y when $x=60$ miles per hour and 120 miles per hour.

   Also compute the coefficient of correlation for the above data. 14

9. It has been found from experience that the mean breaking strength of a particular brand of thread is 275·6 grams with a standard deviation of 39·7 grams. Recently a sample of 36 pieces of thread showed a mean breaking strength of 253·2 grams. Can one conclude at a significance level of *(a)* 0·05 and *(b)* 0·01 that the thread has become inferior? 7+7

Given that $f(t)=\frac{1}{\sqrt{2\pi}}\int_0^t e^{-\frac{x^2}{2}}dx$, then

$f(2\cdot0)=0\cdot4772$, $f(2\cdot41)=0\cdot4920$, $f(1\cdot58)=0\cdot4429$,
$f(0\cdot47)=0\cdot1808$, $f(1\cdot645)=0\cdot45$, $f(1\cdot96)=0\cdot4750$,
$f(0\cdot41)=0\cdot1591$, $f(1\cdot28)=0\cdot40$, $f(1\cdot23)=0\cdot39$,
$f(1\cdot475)=0\cdot43$, $f(2\cdot33)=0\cdot49$, $f(2\cdot58)=0\cdot495$

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