Code : 103102

B.Tech 1st Semester Special Exam., 2020

MATHEMATICS—I

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. **1** *is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) Let $f(x) = x^3 - 6x^2 + 11x - 6$ be a function defined on [1, 3]. Then the point $c \in (1, 3)$ such that $f'(c) = 0$ is given by

(i) $c = 2 \pm \frac{1}{\sqrt{2}}$

(ii) $c = 2 \pm \frac{1}{\sqrt{3}}$

(iii) $c = 2 \pm \frac{1}{2}$

(iv) None of the above

(b) $\sin x$ when expanded in power of $x - \frac{\pi}{2}$ is

(i) $1 + \frac{\left(x-\frac{\pi}{2}\right)^2}{2!} + \frac{\left(x-\frac{\pi}{2}\right)^3}{3!} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!} + \cdots$

(ii) $1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2!} + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!} + \cdots$

(iii) $\left(x-\frac{\pi}{2}\right)^2 + \frac{\left(x-\frac{\pi}{2}\right)^3}{3!} + \frac{\left(x-\frac{\pi}{2}\right)^5}{5!} + \cdots$

(iv) None of the above

(c) Locus of the centre of curvature of a curve is called

(i) envelope of the curve

(ii) involute of the curve

(iii) evolute of the curve

(iv) None of the above

(d) The value of $\Gamma -\frac{3}{2}$ is

(i) $\frac{4\sqrt{\pi}}{3}$

(ii) $-\frac{4\sqrt{\pi}}{15}$

(iii) $\frac{4\pi}{3}$

(iv) $-\frac{4\pi}{3}$

(e) At $x = a$, the function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{x^2}{a} - a, & 0 < x < a \\ 0, & x = a \\ a - \frac{a^3}{x^2}, & x > a \end{cases}$$

has

(i) continuity

(ii) mixed discontinuity

(iii) removable discontinuity

(iv) None of the above

(f) The function $e^x + 2\cos x + e^{-x}$ has minima at $x =$

(i) π

(ii) $\frac{\pi}{2}$

(iii) 0

(iv) None of the above

(g) If P_n is the Legendre polynomial of first kind, then the value of $\int_{-1}^{1} P_{n+1}^2 \, dx$ is

(i) $\frac{2}{(2n+1)}$

(ii) $\frac{2}{(2n+2)}$

(iii) $\frac{2}{(2n+3)}$

(iv) $\frac{2}{(2n+4)}$

(h) The differential equation

$$N(x, y)\,dx + M(x, y)\,dy = 0$$

is an exact differential equation, if

(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(ii) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

(iii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iv) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} = 1$

(i) The integrating factor of the differential equation $x(1+y^2)dy + y(1+x^2)dx = 0$ is

(i) $\frac{1}{x}$

(ii) $\frac{1}{y}$

(iii) xy

(iv) $\frac{1}{xy}$

(j) If F is a conservative force field, then the value of curl F is

(i) 0

(ii) 1

(iii) ∇F

(iv) -1

2. (a) Show that the evolute of a cycloid is another cycloid. 7

(b) Express

$$\int_0^1 x^m (1-x^p)^n \, dx$$

in term of beta function and hence evaluate the integral

$$\int_0^1 x^{3/2} (1-\sqrt{x})^{1/2} \, dx$$ 7

(Turn Over)

3. (a) The function $f(x) = \sin x$ is approximated by Taylor's polynomial of degree three about the point $x = 0$. Find c such that the error satisfies $|R_3(x)| \leq 0 \cdot 001$ for all x in the interval $[0, c]$. 7

(b) Find the Fourier series expansion of the following periodic function of period 4 : 7

$$f(x) = \begin{cases} 2+x, & -2 \leq x \leq 0 \\ 2-x, & 0 \leq x \leq 2 \\ f(x+4) = f(x) & \end{cases}$$

4. (a) Find the power series solution about $x = 0$, of the differential equation $y'' - 4y = 0$. 7

(b) Find the directional derivative of $d(x, y, z) = \sqrt{xy^2 + 2x^2 z}$ at $(2, -2, 1)$ in the direction of negative z-axis. 7

5. (a) Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $x^2/4 + y^2/9 = 1$. 7

(b) Evaluate the integral

$$\int_0^2 \int_0^{y^2/2} \frac{y}{\sqrt{x^2 + y^2 + 1}} \, dx \, dy$$ 7

(Continued)

6. (a) Evaluate the triple integral

$$\iiint_T y\,dx\,dy\,dz$$

where T is the region bounded by the surface $x = y^2$, $x = y + 2$, $4z = x^2 + y^2$ and $z = y + 3$. 7

(b) Use the divergence theorem to evaluate $\iint_S (v \cdot n)\,dA$, where $v = x^2 z i + y j - xz^2 k$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. 7

7. (a) Solve the differential equation

$$(3x^2 y^3 e^y + y^3 + y^2)dx + (x^3 y^3 e^y - xy)dy = 0$$ 7

(b) Obtain the general solution and singular solution of the nonlinear equation $y = xy' + (y')^2$. 7

8. (a) Express $f(x) = x^4 + 2x^3 - 6x^2 + 5x - 3$ in terms of Legendre polynomials. 6

(b) Show that

$$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{1}{x^2}(3 - x^2)\sin x - \frac{3}{x}\cos x\right]$$ 8

(Turn Over)

9. (a) Find the general solution of the partial differential equation

$$(y + z)p + (z + x)q = x + y$$ 7

(b) Find the complete integral of the partial differential equation

$$p^2 q^2 = 9p^2 y^2 (x^2 + y^2) - 9x^2 y^2$$ 7

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