Code : BSC-202 (100312)

B.Tech 3rd Semester Special Exam., 2020

(New Course)

MATHEMATICS—III

(PDE, Probability and Statistics)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

(v) Relevant statistical data are given at the end of the question paper.

1. Choose the correct answer of the following (any *seven*) : 2×7=14

(a) If P_n is the Legendre polynomial, then the value of $\int_{-1}^{1} x^m P_n dx$, $(m < n)$ is

(i) $\frac{2}{(2n+1)}$

(ii) 0

(iii) 1

(iv) $\frac{2}{(2m+1)}$

(b) If J_n is the Bessel's function of first kind, then

$$\left[J_{-\frac{1}{2}}(x)\right]^2 + \left[J_{\frac{1}{2}}(x)\right]^2$$

is equal to

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\frac{2}{\pi x}$

(iv) $\frac{1}{\pi x}$

(c) The particular integral of $(D^2 - a^2 D'^2) Z = x^2$, is

(i) $\frac{1}{12} x^4$

(ii) $\frac{1}{3} x^3 + \frac{1}{2} y x^2$

(iii) $\frac{1}{4} x^4$

(iv) $x^4 + \frac{1}{2} y x^2$

(d) The function $3x^2+5x-6$ in terms of Legendre polynomial is equal to

(i) $P_2+5P_1-5P_0$

(ii) $2P_2+P_1-5P_0$

(iii) $2P_2+P_1-P_0$

(iv) $2P_2+5P_1-5P_0$

(e) Let the joint probability density function of the continuous random variables X and Y be

$$f(x, y)=\begin{cases} k(x^2+y^2); & 0<x<1, 0<y<1 \\ 0 \quad ; & \text{elsewhere} \end{cases}$$

Then the value of k is

(i) 1

(ii) 3/2

(iii) 2

(iv) 5/2

(f) Let A and B be any two events. Which one of the following is correct?

(i) $P(A\cap\overline{B})=P(A)-P(A\cap B)$

(ii) $P(\overline{A}\cap B)=P(A)-P(A\cap B)$

(iii) $P(A\cup B)=P(A)+P(B)$

(iv) $P(A\cap B)=P(A)+P(B)$

(g) If $P(A\cap B)=\frac{1}{4}$, $P(A\cup B)=\frac{3}{4}$, $P(\overline{A})=\frac{2}{3}$, then $P(A/B)$ is equal to

(i) 1/3

(ii) 1/4

(iii) 1/2

(iv) 3/8

(h) If μ is the mean and σ is the standard deviation of a set of measurements, which are normally distributed, then the percentage of measurements within the range $\mu\pm\sigma$ is

(i) 70

(ii) 65

(iii) 67·45

(iv) 68·26

(i) If the density function of gamma distribution is

$$f(x)=\begin{cases} \dfrac{x^{\alpha-1}e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma\alpha}, & x>0 \\ 0 \quad , & x\le 0 \end{cases}$$

Then mean is equal to

(i) α

(ii) β

(iii) $\alpha\beta$

(iv) $\alpha\beta^2$

(j) The moment generating function of a continuous random variable X be given as $M_X(t) = (1-t)^{-5}$ for $|t| < 1$. Then its mean and variance are

(i) $\left(5, \frac{1}{5}\right)$

(ii) $\left(\frac{1}{5}, \frac{1}{5}\right)$

(iii) $\left(5, \frac{1}{15}\right)$

(iv) $(5, 5)$

2. *(a)* Solve :

$$(2xy-1)\frac{\partial z}{\partial x} + (z-2x^2)\frac{\partial z}{\partial y} = 2(x-yz)$$

(b) Solve :

$$(D^2 - DD' - 2D'^2)\,Z = (2x^2 + xy - y^2)\sin xy - \cos xy$$

7+7

3. Reduce the following equation into canonical form and hence solve it : 14

$$x^2(y-1)\frac{\partial^2 z}{\partial x^2} + x(1-y^2)\frac{\partial^2 z}{\partial x \partial y} + y(y-1)\frac{\partial^2 z}{\partial y^2} + xy\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

4. State and prove orthogonal properties of Legendre polynomials. 14

5. *(a)* The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by this treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly?

(b) State axiomatic definition of the probability. Also prove that for any two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

7+7

6. Let the joint probability density function of the continuous random variables X and Y be

$$f(x, y) = \begin{cases} kxy; & 0 < x < 2,\ 1 < y < 3 \\ 0\ ; & \text{elsewhere} \end{cases}$$

Find the value of k and probability density function of $X + Y$. Also find the mean and variance of X and Y. 14

7. *(a)* Fit a second degree parabola to the following data, where x is the independent variable :

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

(b) If X and Y are two uncorrelated variables and if $u = x + y$, $v = x - y$, then find the coefficient of correlation between u and v in terms of σ_x and σ_y the standard deviations of x and y respectively. 7+7

8. (a) The first four moments of a distribution about the value 4 of the variable are $-1 \cdot 5$, 17, -30, 108. Find first four moments about the mean.

(b) In a distribution exactly normal, 17% of the items are under 35 and 79% items are under 63. What are the mean and standard deviation of the distribution? 7+7

9. (a) If the probability that an individual suffers a bad reaction from injection of a given serum is 0·001, determine the probability that out of 1000 individuals (i) exactly 4 and (ii) more than 3 individuals will suffer a bad reaction.

(b) The mean lifetime of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean lifetime of all

the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against the alternative hypothesis $\mu \neq 1600$ using a level of significance of (i) 0·05 and (ii) 0·01. 7+7

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Statistical data :

Given that $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}} dx$, while

$f(0 \cdot 81) = 0 \cdot 29$, $f(0 \cdot 96) = 0 \cdot 33$,
$f(1 \cdot 0) = 0 \cdot 3413$, $f(1 \cdot 230) = 0 \cdot 39$,
$f(1 \cdot 475) = 0 \cdot 43$, $f(1 \cdot 96) = 0 \cdot 4750$,
$f(2 \cdot 0) = 0 \cdot 4772$, $f(2 \cdot 58) = 0 \cdot 4950$,
$f(3 \cdot 0) = 0 \cdot 4987$

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