Code : 105102

B.Tech 1st Semester Exam., 2019
(New Course)

MATHEMATICS—I

(Calculus and Linear Algebra)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct answer of the following (any *seven*) : 2×7=14

(a) Evolute of the parabola $y^2 = 4ax$ is

(i) $ay^2 = 4(x-2a)^2$

(ii) $27ay^2 = (x-2a)^2$

(iii) $y^2 = (x-2a)^2$

(iv) $27ay^2 = 4(x-2a)^2$

(b) The maximum value of

$$f(x) = x^5 - 5x^4 + 5x^3 - 1$$

is

(i) -1

(ii) 0

(iii) 5

(iv) 20

(c) Let P be a nonzero $n \times n$ real matrix with $n \geq 2$, which of the following implications is valid?

(i) Det $(P) = 0$ implies rank $(P) = 0$

(ii) Det $(P) = 1$ implies rank $(P) \neq 1$

(iii) Rank $(P) = 1$ implies det $(P) \neq 0$

(iv) Rank $(P) = n$ implies det$(P) \neq 1$

(d) The power series

$$\sum_0^\infty 2^{-n} x^{2n}$$

is convergent if

(i) $|x| \leq 2$

(ii) $|x| < 2$

(iii) $|x| < \sqrt{2}$

(iv) $|x| \leq \sqrt{2}$

(e) For a real skew-symmetric matrix A of odd order, the determinant of A is equal to

(i) 0

(ii) 1

(iii) 2

(iv) −1

(f) Which of the following is incorrect for the matrices A and B?

(i) $A^2 - B^2 = (A-B)(A+B)$

(ii) $(A^T)^T = A$

(iii) $(AB)^n = A^n B^n$, where A, B commute

(iv) $(A-i)(A+i) = 0 \leftrightarrow A^2 = i$

(g) Let

$$f(x) = |x|^{\frac{3}{2}},\ x \in R$$

then

(i) f is uniformly continuous

(ii) f is continuous, but not differentiable at $x = 0$

(iii) f is differentiable and derivative of f is continuous

(iv) f is differentiable, but derivative of f is discontinuous at $x = 0$

(h) If

$$\varphi(x, y, z) = xz^3 i - 2x^2 yz\, j + 2yz^4 k$$

then $(\nabla \times \varphi)$ at the point $(1, -1, 1)$ is

(i) $3j + 4k$

(ii) $6i - 9j + 4k$

(iii) $6i - 9j - 4k$

(iv) $-12i - 9j + 16k$

(i) The value of $\iint dx\,dy$ over the region

$$x^2 + 4y^2 \leq 4$$

is

(i) π

(ii) 2π

(iii) $\frac{\pi}{4}$

(iv) $\frac{\pi}{2}$

(j) The Fourier series of the periodic function $f(x) = 1$, when $-5 < x < 0$ and $f(x) = 3$, when $0 < x < 5$. At $x = 5$, the series will converge to

(i) 0

(ii) 1

(iii) 2

(iv) 3

2\. (a) Find the evolutes of the hyperbola $2xy = a^2$.

(b) Evaluate the integral

$$\int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\,dy$$

7+7

3\. (a) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about the straight line $y = a$. Find the volume of the solid generated.

(b) Evaluate the following integral

$$\iint (x^2+y^2)\,dy\,dx$$

over the area bounded by the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

7+7

4\. (a) Test the convergence of

$$1+\frac{3}{7}x+\frac{3\cdot 6}{7\cdot 10}x^2+\frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3+\frac{3\cdot 6\cdot 9\cdot 12}{7\cdot 10\cdot 13\cdot 16}x^4+\cdots$$

(b) Examine the convergence of the series of which the general term is

$$\frac{2^2 4^2 6^2 \cdots (2n-2)^2}{3\cdot 4\cdot 5 \cdots\cdots (2n-1)\cdot 2n}x^{2n}$$

7+7

(Turn Over)

5\. (a) Obtain the fourth-degree Taylor's polynomial approximation to $f(x) = e^{2x}$ about $x = 0$. Find the maximum error when $0 \le x \le 0\cdot 5$.

(b) It is given that the Rolle's theorem holds the function $f(x) = x^3 + bx^2 + cx,\ 1 \le x \le 2$ at the point $x = \frac{4}{3}$. Find the values of b and c.

7+7

6\. (a) Compute

$$\frac{\partial^2 f}{\partial x \partial y}(0, 0) \text{ and } \frac{\partial^2 f}{\partial y \partial x}(0, 0)$$

for the function

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x+y^2}, & (x, y) \ne (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Also discuss the continuities of

$$\frac{\partial^2 f}{\partial x \partial y} \text{ and } \frac{\partial^2 f}{\partial y \partial x} \text{ at } (0, 0).$$

(b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$.

7+7

(Continued)

7. *(a)* Find the Fourier coefficients corresponding to the function

$$f(x)=\begin{cases}0, & -5<x<0\\ 3, & 0<x<5\end{cases} \qquad \text{Period} = 10$$

(b) Write the corresponding Fourier series.

(c) How should $f(x)$ be defined at $x=-5$, $x=0$ and $x=5$ in order that the Fourier series will converge to $f(x)$ for $-5\le x\le 5$? 14

8. *(a)* Evaluate $A\times(\nabla\phi)$, where

$$A = yz^2 i - 3xz^2\ j + 2xyz k \text{ and } \phi = xyz$$

(b) Show that matrices A and A^T have the same eigenvalues. 7+7

9. *(a)* Solve completely the system of equations

$$2x-2y+5z+3w=0$$
$$4x-y+z+w=0$$
$$3x-2y+3z+4w=0$$
$$x-3y+7z+6w=0$$

(b) Determine the eigenvalues and eigenvectors of the following matrix :

$$\begin{bmatrix} 6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

7+7

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