Code : 211405

B.Tech 4th Semester Exam., 2019

DISCRETE MATHEMATICAL STRUCTURE AND GRAPH THEORY

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) The statement $p \to q$ is logically equivalent to

(i) $p \vee q$

(ii) $p \vee \sim q$

(iii) $\sim p \vee q$

(iv) $\sim p \to q$

(b) The contrapositive of the conditional statement $p \to q$ is

(i) $q \to p$

(ii) $\sim p \to \sim q$

(iii) $\sim p \to q$

(iv) $\sim q \to \sim p$

(c) If A and B are two ~~nonempty sets~~ having n elements in common, then $A \times B$ and $B \times A$ will have how many elements in common?

(i) 2^n

(ii) n^2

(iii) n^4

(iv) $2n$

(d) If a set A have n elements, then how many relations will be there on set A?

(i) n^2

(ii) 2^{n^2}

(iii) 2^n

(iv) $2n$

(e) If $P(\phi)$ represents the power set of ϕ, then $n(P(P(P(\phi))))$ equal to

(i) 1

(ii) 2

(iii) 3

(iv) 4

(f) For the poset [{3, 5, 9, 15, 24, 45}; divisor of] the bus of {3, 5} is

(i) 3

(ii) 5

(iii) 15

(iv) 45

(g) If $(S, *)$ is a monoid, where $S = \{1, 2, 3, 6\}$ and $*$ is defined by $a * b = \text{lcm}(a, b)$, where $a, b \in S$, then the identity element is

(i) 1

(ii) 2

(iii) 3

(iv) 6

(h) The total number of subgroups of group G of prime order is

(i) 1

(ii) 2

(iii) 3

(iv) 4

(i) The number of edges in a bipartite graph with n vertices is at most

(i) $\frac{n^2}{2}$

(ii) $\frac{n^2}{4}$

(iii) n^2

(iv) $2n$

(j) The number of pendant vertices of a full-binary tree is

(i) $\frac{n+1}{2}$

(ii) $\frac{n-1}{2}$

(iii) $\frac{2n+1}{2}$

(iv) $\frac{2n-1}{2}$

2. *(a)* Using truth table, show that—

(i) $((p \to (q \to r))) \to ((p \to q) \to (p \to r))$ is a tautology;

(ii) $\sim(q \to r) \wedge r \wedge (p \to q)$ is a contradiction.

(b) Obtain the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the statement $(p \to (q \wedge r)) \wedge (\sim p \to (\sim q \wedge \sim r))$. 7+7=14

3. *(a)* For any sets A and B, prove that

(i) $(A \cup B)' = A' \cap B'$;

(ii) $(A \cap B)' = A' \cup B'$.

(b) If two sets A and B have n elements in common, then show that the sets $A \times B$ and $B \times A$ will have 2^n elements in common. 7+7=14

4. *(a)* If R is the relation on the set of positive integers, such that $(a, b) \in R$ if and only if $a^2 + b$ is even, prove that R is an equivalence relation.

(b) Define partition of a set. If the relation R on the set of integers Z is defined by aRb iff $a \equiv b \pmod 4$, find the partition induced by R. 7+7=14

5. *(a)* If R and S be relations on $A = \{1, 2, 3\}$ represented by the matrices

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

find the matrices that represent *(i)* $R \cup S$, *(ii)* $R \cap S$, *(iii)* $R \circ S$, *(iv)* $R - S$, *(v)* R', *(vi)* $R \circ R$ and *(vii)* $R \oplus S$.

(b) Draw the Hasse diagram representing the partial ordering $\{(A, B) : A \subseteq B\}$ on the power set $P(S)$, where $S = \{a, b, c\}$. Find the maximal, minimal, greatest and least elements of the poset. 7+7=14

6. *(a)* Define characteristic function of a set. If A and B are any two subsets of universal set U, then show that—

$$f_{A \cup B}(x) = f_A(x) + f_B(x) - f_{A \cap B}(x),$$
for all $x \in U$

(b) If functions $f, g, h : Z \to Z$ are defined as

$$f(x) = x - 1, \; g(x) = 3x \text{ and}$$
$$h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

Verify that $f \circ (g \circ h) = (f \circ g) \circ h$. 7+7=14

7\. *(a)* Show that every group of order 3 is cyclic.

(b) Prove that the necessary and sufficient condition for a non-empty set H of a group $(G, *)$ to be a subgroup is $a, b \in H \Rightarrow a * b^{-1} \in H$. 7+7=14

8. *(a)* Show that the order of a subgroup of a finite group is a divisor of the order of the group.

(b) Prove that the set S of all real numbers of the form $a + b\sqrt{2}$, where a, b are integers is an integral domain with respect to usual addition and multiplication. 7+7=14

9. *(a)* Define adjacency matrix and incidence matrix of graph G. Draw the graph represented by the adjacency matrix

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(b) Show that a tree with n vertices has $(n-1)$ edges. 7+7=14

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