Code : 101102

B.Tech 1st Semester Special Exam., 2020

MATHEMATICS—I

Time : 3 hours Full Marks : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are **NINE** questions in this paper.*

(iii) *Attempt **FIVE** questions in all.*

(iv) *Question No.* **1** *is compulsory.*

1. Answer the following as directed (any *seven*) : 2×7=14

(a) The series

$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$$

converges, if

(i) $p>0$

(ii) $p<0$

(iii) $p>1$

(iv) $p\leq 1$

(Choose the correct option)

(b) The value of

$$\underset{x\to\pi/2}{\text{Lt}}\frac{\log(\sin x)}{(\pi/2-x)^2}$$

is

(i) zero

(ii) 1/2

(iii) −1/2

(iv) −2

(Choose the correct option)

(c) $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is equal to

(i) 0

(ii) 1

(iii) $\frac{\pi}{4}$

(iv) $\frac{\pi}{2}$

(Choose the correct option)

(d) In terms of beta function

$$\int_0^{\pi/2}\sin^7\theta\sqrt{\cos\theta}\,d\theta=$$

(i) $\frac{1}{4}\beta(4, 3/4)$

(ii) $\frac{1}{2}\beta(4, 3/4)$

(iii) $\frac{1}{4}\beta(4, 4/3)$

(iv) $\frac{1}{2}\beta(4, 4/3)$

(Choose the correct option)

(e) The period of $\cos 3x$ is

(i) $\frac{\pi}{3}$

(ii) $\frac{2\pi}{3}$

(iii) $\frac{3\pi}{2}$

(iv) 2π

(Choose the correct option)

(f) The function $f(x)$ defined in $(-\pi, \pi)$ can be expanded into Fourier series containing both sine and cosine terms.

(Write True or False)

(g) If

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

then the determinant AB has the value

(i) 4

(ii) 8

(iii) 16

(iv) 32

(Choose the correct option)

(h) The sum and product of the eigenvalues of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are

(i) 7 and 5

(ii) 5 and 7

(iii) 12 and 3

(iv) 3 and 12

(Choose the correct option)

(i) Write the statement of Rolle's theorem.

(j) Define the rank of matrix.

2. *(a)* Test the convergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots \infty$$ 7

(b) Prove that

$$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2},$$

if $0 < a < b < 1$ 7

3. *(a)* Prove that

$$\iint_D x^{l-1} y^{m-1} dx\, dy = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m+1)} h^{i+m}$$

where D is the domain $x \geq 0$, $y \geq 0$ and $x + y \leq h$. 7

(b) Obtain Fourier series for the function

$$f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$$

and hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty=\frac{\pi}{8}$$ 7

4. *(a)* Determine the value of p such that the rank of

$$A=\begin{bmatrix}1 & 1 & -1 & 0\\ 4 & 4 & -3 & 1\\ p & 2 & 2 & 2\\ 9 & 9 & p & 3\end{bmatrix}$$

is 3. 7

(b) Use Gauss-Jordan method to find the inverse of the matrix

$$A=\begin{bmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{bmatrix}$$

7

5. *(a)* Find the non-singular matrices P and Q such that

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\\ 3 & 0 & 5 & -10\end{bmatrix}$$

is reduced to normal form. Also find its rank. 8

(b) Solve the given equations by Cramer's rule : 6

$$x+y+z=4$$
$$x-y+z=0$$
$$2x+y+z=5$$

6. *(a)* Verify Cauchy-Hamilton theorem for the matrix

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

and find its inverse. 7

(b) Find the eigenvectors of the matrix

$$\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

Hence reduce

$$6x^2+3y^2+3z^2-2yz+4zx-4xy$$

to a 'sum of squares'. Also write the nature of the matrix. 7

7. *(a)* Let V be the vector space of all polynomials of degree ≤ 3. Determine whether or not the set $S=\{t^3, t^2+t, t^3+t+1\}$ spans V. 6

(b) Let T be a linear transformation defined by

$$T\left[\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right]=\begin{pmatrix}1\\2\\3\end{pmatrix},\ T\left[\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}\right]=\begin{pmatrix}1\\-2\\3\end{pmatrix},$$

$$T\left[\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}\right]=\begin{pmatrix}1\\-2\\-3\end{pmatrix},\ T\left[\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\right]=\begin{pmatrix}-1\\2\\3\end{pmatrix}$$

Find $T\left[\begin{pmatrix}4 & 5\\ 3 & 8\end{pmatrix}\right]$. 8

8. *(a)* Let $T:\mathfrak{R}^3 \to \mathfrak{R}^2$ be a linear transformation defined by

$$T\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}y+z\\y-z\end{pmatrix}$$

Determine the matrix of the linear transformation T, with respect to the ordered basis

$$X=\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

in $\mathfrak{R}^3$ and $Y=\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ in $\mathfrak{R}^2$. 7

(Turn Over)

(b) Solve the following system of equations using Gauss elimination method : 7

$$4x-3y-9z+6w=0$$
$$2x+3y+3z+6w=6$$
$$4x-21y-39z-6w=-24$$

9. *(a)* Find the values of λ for which the equations

$$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$$
$$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$$
$$2x+(3\lambda+1)+3(\lambda-1)z=0$$

are consistent, and find the ratios of $x:y:z$ when λ has the smallest of these values. What happens when λ has the greatest of these values? 7

(b) Expand $e^{\sin x}$ by Maclaurin's series or otherwise up to the term containing x^4. 7

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