Code : 211303

B.Tech 3rd Semester Exam., 2017

MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Choose the correct option (any *seven*) : 2×7=14

(a) If P_n is the Legendre polynomial of first kind, then the value of

$$\int_{-1}^{1} P_{n+1}^2 \, dx$$

is

(i) $\frac{2}{(2n+1)}$ *(ii)* $\frac{2}{(2n+2)}$

(iii) $\frac{2}{(2n+3)}$ *(iv)* $\frac{2}{(2n+4)}$

(b) If J_n is the Bessel's function of first kind, then the value of $J_{\frac{3}{2}}$ is

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\sqrt{\frac{2}{\pi x}} \sin x$

(iv) $\sqrt{\frac{2}{\pi x}} \cos x$

(c) The general solution of

$$\frac{d^2y}{dx^2} + 9y = \sin^3 x$$

is

(i) $y = A\cos(3x+B) + \frac{1}{24}\sin x - \sin 3x$

(ii) $y = Ae^{3x} + Be^{-3x} + \frac{1}{32}\sin x + \frac{1}{2}\cos 3x$

(iii) $y = A + Be^{3x} + 2\sin x - \frac{5}{13}\sin 3x$

(iv) $y = A\sin(3x+B) + \frac{3}{32}\sin x + \frac{x}{24}\cos 3x$

(d) The general solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is

(i) $u = f(x+iy) + g(x-iy)$

(ii) $u = f(x+y) + g(x-y)$

(iii) $u = cf(x-iy)$

(iv) $u = cg(x+iy)$

(e) The Fourier series of the periodic function $f(x) = x + x^2$, $-\pi < x \leq \pi$ at $x = \pi$ converges to

(i) π

(ii) 2π

(iii) π^2

(iv) $\pi + \pi^2$

(f) The radius of convergence of the series

$$\sum_{n=0}^{\infty} (3+4i)^n z^n$$

is

(i) 5

(ii) 1/5

(iii) 3+4i

(iv) None of the above

(g) The value of the integral

$$\oint_{|z|=2} \frac{e^{2z}}{(z+1)^4} dz$$

is

(i) $2\pi i e^{-1}$

(ii) $\frac{8\pi i}{3} e^{-2}$

(iii) $\frac{2\pi i}{3} e^{-2}$

(iv) 0

(h) Let A, B and C be any three independent events. Which one of the following is incorrect?

(i) $P(A/B) = P(A)$

(ii) $P(B/A) = P(B)$

(iii) $P(A \cap B) = P(A)\,P(B)$

(iv) $P(A \cup B) = P(A) + P(B)$

(i) A random variable X has a Poisson distribution. If

$$4\{P(X=2)\} = \{P(X=1) + P(X=0)\}$$

then the variance of X is

(i) 3

(ii) 2

(iii) 1

(iv) 4

(j) The moment-generating function of a continuous random variable X be given as

$$M_X(t) = (1-t)^{-9} \quad |t| < 1$$

then its mean and variance are

(i) (9, 1/9)

(ii) (9, 9)

(iii) (3, 3)

(iv) (1/9, 1/9)

2. State and prove Rodrigues' formula. 14

3. If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, then find $f(z)$ in terms of z. 14

4. Evaluate

$$\int_0^\infty \frac{\sin(mx)}{x(x^2+a^2)} dx$$

by Contour integration. 14

5. Classify and reduce the equation given below into normal form and find its solution. 14

$$xy\left(\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2}\right) + (x^2 - y^2)\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial x} y - \frac{\partial z}{\partial y} x + 2(x^4 + y^4)$$

6. Show that the solution of the heat equation $u_t = u_{xx}$ satisfying the conditions 14

(i) $u \to 0$, as $t \to \infty$

(ii) $u = 0$, when $x = \pm a$ $\forall$ values of $t > 0$

(iii) $u = x$, when $t = 0$ and $-a < x < a$ $\forall$ values of $t > 0$

is

$$u = \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin\left(\frac{n\pi x}{a}\right) e^{\frac{-n^2\pi^2}{a^2}t}$$

7. (a) If A, B and C are independent events, show that A and $B \cup \overline{C}$ are also independent.

(b) Urn I contains 2 white and 3 black balls, Urn II contains 4 white and 1 black balls and Urn III contains 3 white and 4 black balls. An Urn is selected at random and a ball drawn at random is found to be white. What is the probability that Urn I was selected?

7+7=14

8. (a) The incidence of occupational disease in an industry is such that the workers have a 25% chance of suffering from it. What is the probability that out of 13 workers chosen at random, six or more will suffer from the disease?

(b) In a distribution exactly normal, 7% of the items are under 35 and 89% items are under 63. What are the mean and standard deviation of the distribution? It is given that

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{\frac{-x^2}{2}} dx$$

then $f(1 \cdot 230) = 0 \cdot 39$, $f(1 \cdot 475) = 0 \cdot 43$.

7+7=14

9. A random variable X has the density function

$$f(x) = \begin{cases} cx^2, & 1 \le x \le 2 \\ cx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find—

(i) the constant c

(ii) $P(1 < x < 2)$

(iii) $P(x \ge 3)$

(iv) $P(x < 1)$

(v) $E(2X)$

(vi) $\text{Var}(X - 3)$

(vii) $E(2X + 4)$

2×7=14

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