Code : 102202

B.Tech 2nd Semester Exam., 2019

MATHEMATICS—II

(Ordinary Differential Equations and Complex Variables)

(New Course)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

(iv) Question No. 1 is compulsory.

1. Answer the following (any *seven*) : 2×7=14

(a) Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2i - j - 2k$.

(b) Evaluate $\nabla.[r\,\nabla(1/r^3)]$.

(c) What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + \left(\frac{d^3y}{dx^3}\right)^{3/2} = 0 \ ?$$

(d) Find the general solution of the differential equation

$$x(x^2 + 3y^2)\,dx + y(y^2 + 3x^2)\,dy = 0$$

(e) Evaluate the integral

$$\int_C \frac{(e^z + \sin \pi z)\,dz}{(z-1)(z+1)(z+4)}, \quad C : |z| = 2$$

(f) Evaluate the integral

$$\int_C \frac{dz}{(z^2 + 4z + 3)^2}, \quad C : |z| = 4$$

(g) Define the pole-type singularity with an example.

(h) Find the bilinear transformation that maps $z_1 = \infty$, $z_2 = i$ and $z_3 = 0$ into the points $w_1 = 0$, $w_2 = i$ and $w_3 = \infty$.

(i) If $a < b$, then evaluate the integral

$$\int_a^b |(x-a) + (x-b)|\,dx$$

(j) Evaluate the integral

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$$

2. *(a)* Evaluate the integral

$$\int_0^a \int_y^a \frac{x}{(x^2 + y^2)^2}\,dy\,dx$$

(b) Find the mass of a plate in the form of a quadrant of an ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

whose density per unit area is given by $\rho = kxy$. 7+7=14

3. Evaluate $\int_C F \cdot dr$, where

$$F=(3x^2+6y)i-14yzj+20xz^2k$$

from (0, 0, 0) to (1, 1, 1) along the following paths : http://www.akubihar.com 7+7=14

(a) $x=t,\ y=t^2$ and $z=t^3$

(b) The straight line joining (0, 0, 0) to (1, 1, 1)

4. Solve the following differential equations : 7+7=14

(a) $(x^2+y^2+x)dx-(2x^2+2y^2-y)dy=0$

(b) $y=2px+y^2p^3$

5. (a) State and prove Rodrigues' formula.

(b) Show that

$$2nJ_n(x)=x[J_{n+1}(x)+J_{n-1}(x)]$$ 7+7=14

6. Find the series solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+6x\frac{dy}{dx}+(x^2+6)y=0$$ 14

7. (a) State and prove the sufficient condition for a function $w=f(z)$ to be analytic.

(b) Find an analytic function $f(z)$ such that $\text{Re}\{(f'(z)\}=3x^2-4y-3y^2$ and $f(1+i)=0$. 7+7=14

8. (a) Discuss the nature of the singularities for $\left(\frac{1-\cosh z}{z^3}\right)$. Also determine the order of the pole and corresponding residue if it exists.

(b) Find what regions of the w-plane correspond by the transformation $w=\left(\frac{z-i}{z+i}\right)$ to the interior of a circle of centre $z=-i$. 7+7=14

9. (a) Evaluate

$$\oint_C \frac{\sin^2 z}{z(z-1)(2z+5)}dz,\quad C:|z-1|+|z+1|=3$$

(b) Evaluate

$$\int_0^\infty \frac{\sin(mx)}{x(x^2+a^2)}dx$$ 7+7=14

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