

Code : 102202

(2)

B.Tech 2nd Semester Exam., 2019

MATHEMATICS-II

(Ordinary Differential Equations and
Complex Variables)

(New Course)

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. 1 is compulsory.

1. Answer the following (any seven) : $2 \times 7 = 14$

(a) Find the directional derivative of $\varphi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

~~(b)~~ Evaluate $\nabla \cdot [r \nabla (1/r^3)]$.

~~(c)~~ What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3} \right)^{2/3} + \left(\frac{d^3y}{dx^3} \right)^{3/2} = 0 ?$$

~~(d)~~ Find the general solution of the differential equation

$$x(x^2 + 3y^2)dx + y(y^2 + 3x^2)dy = 0$$

~~(e)~~ Evaluate the integral

$$\int_C \frac{(e^z + \sin \pi z) dz}{(z-1)(z+1)(z+4)}, C: |z|=2$$

~~(f)~~ Evaluate the integral

$$\int_C \frac{dz}{(z^2 + 4z + 3)^2}, C: |z|=4$$

~~(g)~~ Define the pole-type singularity with an example.

~~(h)~~ Find the bilinear transformation that maps $z_1 = \infty$, $z_2 = i$ and $z_3 = 0$ into the points $w_1 = 0$, $w_2 = i$ and $w_3 = \infty$.

~~(i)~~ If $a < b$, then evaluate the integral

$$\int_a^b |(x-a)+(x-b)| dx$$

~~(j)~~ Evaluate the integral

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$$

2. ~~(a)~~ Evaluate the integral

$$\int_0^a \int_y^a \frac{x}{(x^2 + y^2)^2} dy dx$$

(3)

- ~~(b)~~ Find the mass of a plate in the form of a quadrant of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

whose density per unit area is given by
 $\rho = kxy.$

7+7=14

3. Evaluate $\int_C F \cdot dr$, where

$$F = (3x^2 + 6y)i - 14yzj + 20xz^2k$$

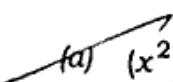
from $(0, 0, 0)$ to $(1, 1, 1)$ along the following paths : <http://www.akubihar.com> 7+7=14

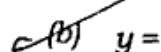
(a) $x = t, y = t^2$ and $z = t^3$

- (b) The straight line joining $(0, 0, 0)$ to $(1, 1, 1)$

 Solve the following differential equations :

7+7=14

 (a) $(x^2 + y^2 + x)dx - (2x^2 + 2y^2 - y)dy = 0$

 (b) $y = 2px + y^2 p^3$

5. (a) State and prove Rodrigues' formula.

- (b) Show that

$$2nJ_n(x) = x[J_{n+1}(x) + J_{n-1}(x)] \quad 7+7=14$$

(4)

- ~~6.~~ Find the series solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} + 6x \frac{dy}{dx} + (x^2 + 6)y = 0$$

14

7. (a) State and prove the sufficient condition for a function $w = f(z)$ to be analytic.

- (b) Find an analytic function $f(z)$ such that $\operatorname{Re}\{f'(z)\} = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0.$ 7+7=14

8. (a) Discuss the nature of the singularities for $\left(\frac{1-\cosh z}{z^3}\right).$ Also determine the order of the pole and corresponding residue if it exists.

- (b) Find what regions of the w -plane correspond by the transformation $w = \left(\frac{z-i}{z+i}\right)$ to the interior of a circle of centre $z = -i.$ 7+7=14

9. (a) Evaluate

$$\int_C \frac{\sin^2 z}{z(z-1)(2z+5)} dz, \quad C: |z-1| + |z+1| = 3$$

- (b) Evaluate

$$\int_0^\infty \frac{\sin(mx)}{x(x^2 + a^2)} dx$$

7+7=14