

## BCA 1st Semester Exam., 2021

## BASIC MATHEMATICS

Time : 3 hours

Full Marks : 60

## Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **SEVEN** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question Nos. 1 and 2 are compulsory.

1. Choose the correct answer (any six) :  $2 \times 6 = 12$ 

(a) If  $x$  and  $y$  are real numbers, then the value of

$$\max(x, y) + \min(x, y)$$

is

- (i)  $x$
- (ii)  $y$
- (iii)  $x/y$
- (iv) None of the above

(b) Let  $A = [-2]$ . Then  $\det(A)$ , i.e.,  $|A|$  is

- (i) 2
- (ii) 1
- (iii) 0
- (iv) None of the above

(c) For a set  $A$ , the power set of  $A$  is denoted by  $2^A$ . If  $A = \{5, \{6\}, \{7\}\}$ , which of the following are true?

1.  $\emptyset \in 2^A$
2.  $\emptyset \subseteq 2^A$
3.  $\{5, \{6\}\} \in 2^A$
4.  $\{5, \{6\}\} \subseteq 2^A$

- (i) 1 and 3 only
- (ii) 2 and 3 only
- (iii) 1, 2 and 3 only
- (iv) 1, 2 and 4 only

(d) The number of positive integers lying between 1 and 100 (both inclusive) and not divisible by 2, 3 and 5 is

- (i) 30
- (ii) 90
- (iii) 95
- (iv) None of the above

( 3 )

(e) If  $Mdx + Ndy = 0$  has the form  $f(y)dx + g(x)dy = 0$ , the integrating factor is

(i)  $1/(Mx - Ny)$

(ii)  $1/(Mx + Ny)$

(iii)  $1/((Mx - Ny) \neq 0)$

(iv) None of the above

(f) In a set of people, the relation 'x is not older than y' satisfies which property?

✓ (i) Transitivity

(ii) Reflexivity

✓ (iii) Antisymmetric

(iv) Symmetric

(g) The pair of equations  $x + 2y + 5 = 0$  and  $-3x - 6y + 1 = 0$  has

(i) a unique solution

(ii) exactly two solutions

(iii) infinitely many solutions

✓ (iv) no solution

( 4 )

(h) The propositional statement  $(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$  is

(i) satisfiable but not valid

(ii) valid

(iii) a contradiction

(iv) None of the above

(i) In an examination, a student scores 4 marks for every correct answer and loses 1 mark for every wrong answer. If he attempts all 75 questions and secures 125 marks, the number of questions he attempted correctly is

(i) 35

✓ (ii) 40

(iii) 42

(iv) 46

(j) If  $f(x) + f(1 - x) = 1$ , then

$$f(1/1997) + f(2/1997) + \dots + f(1996/1997)$$

is

(i) 999

(ii) 998

(iii) 919

(iv) 918

2. Answer any three of the following :  $4 \times 3 = 12$

(a) One kind of bacteria  $y$  grows according to the equation  $\frac{dy}{dt} = ky$ , where  $k$  is a constant and  $t$  is measured in years. If the amount of the bacteria doubles every 5 days, then find the value of  $k$ .

(b) Let  $A$  and  $B$  be sets and let  $A^c$  and  $B^c$  denote the complements of the sets  $A$  and  $B$ . Simplify the expression

$$(A - B) \cup (B - A) \cup (A \cap B)$$

(c) Let  $A = \{1, 2, 3, 4\}$ . Make the relation  $(R)$  over  $A \times A$  as  $R = \{(a, b) \mid a + b > 2 \text{ and } a, b \in A\}$ . Is it transitive?

(d) Check whether  $a \vee b \rightarrow b \wedge c$  is tautology or not.

(e) Find the value of  $\int_0^3 e^x dx$ .

3. Find the value of the integral

$$\int_0^6 \frac{1}{1+x^2} dx \quad 12$$

4. Find the number of subsets of  $\{1, 2, \dots, n\}$  with odd cardinality. 12

5. If  $x = t^2 - 1$  and  $y = 2e^t$ , then find  $\frac{dy}{dx}$ . 12

6. Find the area enclosed by the curve  $y = f(x)$  defined parametrically as

$$x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2} \quad 12$$

7. Find the fourth derivative of  $x^3 \log x$  with respect to  $x$ , using Leibnitz theorem. 12

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