Code : 102102

B.Tech 1st Semester Exam., 2018 (New)

MATHEMATICS—I

(Calculus and Linear Algebra)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Answer/Choose the correct option of the following (any *seven*) : 2×7=14

(a) The evolute of a cycloid is

(i) circle

(ii) another cycloid

(iii) an ellipse

(iv) None of the above

(b) The value of improper integral $\int_0^\infty \sqrt{x}e^{-x^2}dx$ is

(i) $\sqrt{\pi}$

(ii) $\frac{\sqrt{\pi}}{2}$

(iii) $\sqrt{\frac{3}{8}}$

(iv) $\frac{1}{2}\sqrt{\frac{3}{4}}$

(c) If the Cauchy mean value theorem is applicable for the function $f(x)=\frac{1}{x^2}$, $g(x)=\frac{1}{x}$, in $[a, b]$, then the value of c is

(i) $\frac{a+b}{2}$

(ii) $\sqrt{ab}$

(iii) $\frac{2ab}{a+b}$

(iv) None of the above

(d) The value of $\lim_{x\to 0} \sin x \log x$ is

(i) 0

(ii) 1

(iii) −1

(iv) None of the above

(e) Radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ is

(i) 1

(ii) −1

(iii) 0

(iv) ∞

(f) Define half-range sine and cosine series.

(g) $\lim_{x \to 2} \frac{1}{(x-2)^2}$ is

(i) $+\infty$

(ii) $-\infty$

(iii) Exists finitely

(iv) Does not exist

(h) If $u = \sin^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(i) $\frac{2x}{\sqrt{y^2 - x^2}}$

(ii) $\frac{2xy}{x^2 + y^2}$

(iii) 0

(iv) None of the above

(i) Let $M = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix}$. Then

(i) M is diagonalizable but not M^2

(ii) M^2 is diagonalizable but not M

(iii) both M and M^2 are diagonalizable

(iv) neither M nor M^2 is diagonalizable

(j) The possible set of eigenvalues of a 4×4 skew-symmetric orthogonal real matrix is

(i) $\{\pm i\}$

(ii) $\{\pm i, \pm 1\}$

(iii) $\{\pm 1\}$

(iv) $\{0, \pm i\}$

2. (a) Find the evolute of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$. 7

(b) Prove that $1.3.5. \ldots (2n-1) = \frac{2^n \Gamma\left(n + \frac{1}{2}\right)}{\sqrt{\pi}}$. 7

3. (a) Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$. 7

(b) Find the area of the region enclosed by the curve $x=a(t-\sin t)$, $y=a(1-\cos t)$, $0 \le t \le 2\pi$. 7

4. (a) Verify Rolle's theorem for $f(x)=e^{-x}\sin x$ in $(0, \pi)$. 6

(b) Obtain the Taylor's polynomial expansion of the function $f(x)=\sin x$ about the point $x=\frac{\pi}{4}$. Show that the error term tends to zero as $n \to \infty$ for any real x. Hence, write the Taylor's series expansion of $f(x)$. 8

5. (a) A figure consists of a semicircle with a rectangle on its diameter. Given that the perimeter of the figure is 20 metres. Find its dimensions in order that its area may be maximum. 7

(b) Discuss the convergence of the geometric series $\sum_{n=0}^{\infty} r^n$, where r is any real number. 7

(Turn Over)

6. (a) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $xyz=a^3$. 7

(b) Find the directional derivative of $f(x, y)=x^2y^3+xy$ at point (2, 1) in the direction of a unit vector, which makes an angle $\frac{\pi}{3}$ with the x-axis. 7

7. (a) Prove that $\operatorname{div}(fv)=f\operatorname{div}(v)+(\operatorname{grad} f)\cdot v$, where f is a scalar function. 7

(b) Show that the function

$$f(x, y)=\begin{cases}(x+y)\sin\left(\frac{1}{x+y}\right), & x+y\neq 0\\ 0 & , x+y=0\end{cases}$$

is continuous at (0, 0) but its partial derivatives f_x and f_y do not exist at (0, 0). 7

8. (a) Find the rank of the matrix

$$\begin{bmatrix} 2 & 3 & 4 & -1\\ 5 & 2 & 0 & -1\\ -4 & 5 & 12 & -1\end{bmatrix}.$$ 7

(b) For what values of λ and μ do the system of equations $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$ have *(i)* no solution, *(ii)* unique solution and *(iii)* more than one solution? 7

(Continued)

9. *(a)* Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 8

(b) Show that the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. Then, obtain the matrix $B = A^2 + 5A + 3I$. 6

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