Code : 211202

B.Tech 2nd Semester Exam., 2018

MATHEMATICS—II

Time : 3 hours

Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct answer (any *seven*) : 2×7=14

(a) The Fourier series of the periodic function

$$f(x) = x + x^2, -\pi < x \le \pi$$

at $x = \pi$ converges to

(i) π

(ii) 2π

(iii) π^2

(iv) $\pi + \pi^2$

(b) The radius of convergence of the series

$$\sum_{n=0}^{\infty} (3+4i)^n z^n$$

is

(i) 5

(ii) $\frac{1}{5}$

(iii) $3+4i$

(iv) None of the above

(c) The Laplace transform of

$$3 \cosh(5t) - 4 \sinh(5t)$$

is

(i) $\frac{(3s-20)}{(s^2-25)}, s > 5$

(ii) $\frac{(3s-20)}{(s^2-25)}, s < 5$

(iii) $\frac{(3s-20)}{(s^2+25)}, s > 5$

(iv) $\frac{(3s-20)}{(s^2+25)}, s < 5$

(d) If $\delta(t)$ is the Dirac delta function, than $L(\delta t)$ is

(i) -1

(ii) 0

(iii) 1

(iv) None of the above

(e) If L is the Laplace operator, then $L^{-1}\left(s^{-\frac{3}{2}}\right)$ is

· (i) $\sqrt{\frac{t}{\pi}}$

(ii) $\sqrt{\frac{\pi}{t}}$

(iii) $2\sqrt{\frac{\pi}{t}}$

(iv) $2\sqrt{\frac{t}{\pi}}$

(f) If $\varphi(x, y, z) = 3x^2y - y^3z^2$, then $\nabla\varphi$ at the point (1, 1, –2) is

(i) $12i - 9j - 16k$

· (ii) $6i - 9j + 4k$

(iii) $6i - 9j - 4k$

(iv) $-12i - 9j + 16k$

(g) If $a < b$, then $\int_a^b |(x-a)+(x-b)|\,dx$ is equal to

· (i) $\frac{(b-a)^2}{2}$

(ii) $\frac{(b^2-a^2)}{2}$

(iii) $\frac{(a^3-b^3)}{2}$

(iv) $(b-a)^2$

(h) The value of $\iint x^2y^2\,dxdy$ over the region $x^2+y^2 \le 1$ is

(i) $\frac{\pi}{6}$

(ii) $\frac{\pi}{12}$

(iii) $\frac{\pi}{24}$

· (iv) $\frac{\pi}{48}$

(i) If $A(2) = 2i - j + 2k$, $A(3) = 4i - 2j + 3k$, then $\int_2^3 A \cdot \frac{dA}{dt}\,dt$ is

(i) 5

(ii) 10

(iii) 15

(iv) 20

(j) If $\vec{A}, \vec{B}, \vec{C}$ are three mutually perpendicular vectors, each of magnitude unity, then $\left|\vec{A}+\vec{B}+\vec{C}\right|$ is equal to

(i) $\sqrt{3}$

· (ii) 3

(iii) $\sqrt{2}$

(iv) 2

2. (a) Test the convergence of

$$1+\frac{3}{7}x+\frac{3\cdot 6}{7\cdot 10}x^2+\frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3+\frac{3\cdot 6\cdot 9\cdot 12}{7\cdot 10\cdot 13\cdot 16}x^4+\cdots$$

(b) Examine the convergence of the series of which the general term is

$$\frac{2^2\,4^2\,6^2\ldots.(2n-2)^2}{3\cdot 4\cdot 5\ldots\ldots(2n-1)\cdot 2n}x^{2n}$$

7+7=14

3. (a) State and prove the convergence theorem for Laplace transform.

(b) Find

$$L^{-1}\left(\frac{1}{(s^2+1)^2(s^2+4)}\right)$$

using the convolution theorem. 7+7=14

4. (a) Solve the differential equation

$$\frac{d^2y}{dx^2}+9y=\cos 2t,\quad y(0)=1,\quad y\left(\frac{\pi}{2}\right)=-1$$

using Laplace transform.

(b) Evaluate the integral

$$\int_0^\infty \frac{e^{-t}\sin t}{t}dt$$

using Laplace transform. 7+7=14

5. Expand in Fourier series

$$f(x)=\begin{cases}-x, & -4\le x\le 0\\ x, & 0\le x\le 4\end{cases}$$

and hence deduce that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+\cdots$$

14

6. (a) Evaluate by changing the order of integration

$$\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dy\,dx$$

(b) Find the volume of the solid in the first octant bounded by the paraboloid

$$z=36-4x^2-9y^2$$

7+7=14

7. (a) Find the mass of a plate in the form of a quadrant of an ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

whose density per unit area is given by $\rho = kxy$.

(b) Evaluate the following integral by changing to polar coordinates :

$$\int_0^1\int_0^{\sqrt{(2x-x^2)}}(x^2+y^2)\,dy\,dx$$

7+7=14

8. (a) Use divergence theorem to evaluate

$$\iint_S A \cdot d\vec{S},$$

where

$$A = 4xi - 2y^2 j + z^2 k$$

and S is the surface, $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

(b) Find the directional derivative of

$$\varphi(x, y, z) = x^2 yz + 4xz^2$$

at $(1, -2, -1)$ in the direction $2i - j - 2k$.

7+7=14

9. (a) Evaluate

$$\oint_C (x^2 - 2xy)\,dx + (x^2 y + 3)\,dy$$

around the boundary of the region defined by $y^2 = 8x$ and $x = 2$ using Green's theorem.

(b) Find *(i)* $A \times (\nabla\varphi)$ and *(ii)* $(\nabla \times A) \times B$, where

$\vec{A} = x^2 z\, i + y z^3 j - 3xy k$, $\vec{B} = 3x i + 4z j - xy k$

and $\varphi = xy^2 z$. 7+7=14

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