Code : BS-101 (104303)

B.Tech 3rd Semester Special Exam., 2020

MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions:

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

(iv) Question No. 1 is compulsory.

1. Answer the following as directed (any *seven*) : 2×7=14

(a) The value of Chebyshev polynomials $T_3(x)$ is

(i) $2x^2 - 1$

(ii) $2x^2 + 1$

(iii) $4x^3 + 3x$

(iv) $4x^3 - 3x$

(b) The value of $\frac{d}{dx}(xbei'x)$ is

(i) $x\,ber\,x$

(ii) $-x\,ber\,x$

(iii) $-bei\,x$

(iv) $-x\,ber'x$

(c) Which of the following functions is inverse of itself ?

(i) $f(x) = \frac{1-x}{1+x}$

(ii) $f(x) = 5^{\log x}$

(iii) $f(x) = 2^{x(x-1)}$

(iv) None of the above

(d) Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x + x^3}{1 - 3x^2}$. Then $f \circ g(x)$ equals

(i) $-f(x)$

(ii) $3f(x)$

(iii) $[f(x)]^3$

(iv) None of the above

(e) If a function $f : [2, \infty) \to B$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then $B =$

(i) R

(ii) $[1, \infty)$

(iii) $[4, \infty)$

(iv) $[5, \infty)$

(f) If r_1 and r_2 are two regression coefficients, then signs of r_1 and r_2 depend on _____.

(Fill in the blank)

(g) Regression coefficient of y on x is 0·7 and that of x on y is 3·2. Is the correlation coefficient r consistent?

(State Yes or No)

(h) The SD of the binomial distribution is

(i) $\sqrt{npq}$

(ii) $\sqrt{np}$

(iii) npq

(iv) pq

(i) A hypothesis is true but is rejected, then there is an error of type _____.

(Fill in the blank)

(j) A hypothesis is false but is accepted, then there is an error of type _____.

(Fill in the blank)

2. (a) State and prove the orthogonality property of Chebyshev polynomials. 7

(b) Show that

$$(1 - x^2)T_n'(x) = nT_{n-1}(x) - nxT_n(x)$$

where $T_n(x)$ is Chebyshev polynomials. 7

3. (a) What are the applications of wavelets transform? 7

(b) For any three sets A, B and C, prove that

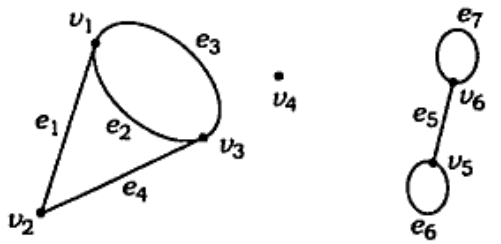
$$A \times (B \cup C) = (A \times B) \cap (A \times C)$$ 7

4. (a) Prove that the relation R on the set $N \times N$ defined by $(a, b)R(c, d) \in R \Leftrightarrow a + d = b + c$ for all (a, b), $(c, d) \in N \times N$ is an equivalence relation. 7

(b) Show that the function $f : R \to R$ defined by $f(x) = 3x^3 + 3$ for all $x \in R$ is a bijection. 7

5. (a) Derive the expression of ber and bei functions. 7

(b) Consider the following graph :



Find all edges that are incident on v_1, all vertices that are adjacent to v_1, all edges that are adjacent to e_1, all loops, all parallel edges, all vertices that are adjacent to themselves, and all isolated vertices. 7

6. (a) The first four moments about the working mean 28·5 of a distribution are 0·294, 7·144, 42·409 and 454·98. Calculate the moment about the mean. Also evaluate β_1, β_2 and comment upon the skewness and kurtosis of the distribution. 7

(b) Compute the Karl Pearson's coefficient correlation using the data given below : 7

X	:	1	5	3	2	1	1	7	3
Y	:	6	1	0	0	1	2	1	5

7. (a) Find the moment-generating function of Poisson distribution. 7

(b) In a certain factory turning out razor blades, there is a small chance of 0·002 for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10000 packets. 7

8. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and SD of the distribution. 7

(b) Discuss the method of least square. 7

9. (a) The mean of a certain normal population is equal to the standard error of the mean of the samples of 100 from that distribution. Find the probability that the mean of the sample of 25 from the distribution will be negative. 7

(b) An unbiased coin is thrown n times. It is desired that the relative frequency of the appearance of head should lie between 0·49 and 0·51. Find the smallest value of n that will ensure this result with 90% confidence. 7

Note : Please provide the distribution table to students.

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