

BCA 1st Semester Exam., 2018

BASIC MATHEMATICS

Time : 3 hours

Full Marks : 60

Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **SEVEN** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question Nos. 1 and 2 are compulsory.

1. Choose the correct answer (any six) : $2 \times 6 = 12$

(a) A function $f(x)$ is continuous in the interval $[0, 2]$. It is known that $f(0) = f(2) = -1$ and $f(1) = 1$. Which one of the following statements must be true?

- (i) There exists a y in the interval $(0, 1)$ such that $f(y) = f(y+1)$.
- (ii) For every y in the interval $(0, 1)$, $f(y) = f(2-y)$.
- (iii) The maximum value of the function in the interval $(0, 2)$ is 1.
- (iv) There exists a y in the interval $(0, 1)$ such that $f(y) = -f(2-y)$.

(Turn Over)

(b) Let $P(A)$ denotes the power set of A . If $P(A) \subseteq B$, then

(i) $2^{|A|} \leq |B|$

(ii) $2^{|A|} \geq |B|$

(iii) $2^{|A|} < |B|$

(iv) None of the above

(c) The power set of countably infinite set is

(i) countable

(ii) uncountable

(iii) None of the above

(d) What are all values of k for which the graph of $y = x^3 - 3x^2 + k$ will have three distinct x -intercepts?

(i) All $k > 0$ (ii) All $k < 4$ (iii) $k = 0, 4$

(iv) None of the above

(e) What is the average of all multiples of 10 from 2 to 198?

(i) 90

(ii) 100

(iii) 110

(iv) 120

(f) The value of $(x-a)(x-b)(x-c)\cdots(x-z)$ is

(i) $x^n - x(a+b+c+\cdots) + \cdots$

(ii) 0

(iii) 1

(iv) None of the above

(g) If x is real and $|x^2 - 2x + 3| = 11$, then the possible values of $|-x^3 + x^2 - x|$ include

(i) 2, 4

(ii) 2, 14

(iii) 4, 52

(iv) 14, 52

(h) A non-zero polynomial $f(x)$ of degree 3 has roots at $x = 1$, $x = 2$ and $x = 3$. Which one of the following must be true?

(i) $f(0)f(4) < 0$

(ii) $f(0)f(4) > 0$

(iii) $f(0) + f(4) > 0$

(iv) $f(0) + f(4) < 0$

(i) Let R be the relation on the set of positive integers such that aRb if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is true?

(i) R is symmetric and reflexive but not transitive

(ii) R is reflexive but not symmetric and not transitive

(iii) R is transitive but not reflexive and not symmetric

(iv) R is symmetric but not reflexive and not transitive

(j) The cardinality of the power set of $\{0, 1, 2, \dots, 10\}$ is

(i) 1024

(ii) 2048

(iii) 10

(iv) 11

2. Answer any three of the following : $4 \times 3 = 12$

(a) Find the number of positive integers lying between 1 and 100 (both inclusive) but not divisible by 2, 3 and 5.

(b) Prove that, for all sets A, B and C , $(A - B) \cap (C - B) = (A \cap C) - B$.

(c) If $y = (t^2 + 2)^2$ and $t = x^{1/2}$, then determine dy/dx .

(d) At $t = 0$, a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(e) If $F(x) = \int_0^x e^{-t^2} dt$, then find $F'(x)$.

3. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t . At $t = 0$, the number of bacteria was present 7500. Find the number present at $t = 5$. 12

4. If n is a known positive integer, then for what value of k is $\int_1^k x^{n-1} dx = \frac{1}{n}$? 12

5. If $y = e^{nx}$, then find $d^n y / dx^n$. 12

6. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$? 12

7. The line segment connecting $(x, 6)$ and $(9, y)$ is bisected by the point $(7, 3)$. Find the values of x and y . 12
