Code : 211405

B.Tech 4th Semester Exam., 2016

DISCRETE MATHEMATICAL STRUCTURES

Time : 3 hours   Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are **NINE** questions in this paper.

(iii) Attempt **FIVE** questions in all.

(iv) Question No. **1** is compulsory.

1. Choose and write the correct option (any *seven*) :   2×7=14

   (a) Let f and g be the functions defined by $f(x) = 2x + 3$ and $g(x) = 3x - 2$, then composition of f and g is

   (i) $6x + 6$

   (ii) $5x + 5$

   (iii) $6x + 7$

   (iv) $7x + 5$

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(2)

(b) Among 200 people, 150 either swim or jog or both. If 85 swim and 60 swim and jog, how many jog?

(i) 125

(ii) 225

(iii) 85

(iv) 25

(c) A graph in which all nodes are of equal degree is known as _____ graph.

(i) complete

(ii) multi-

(iii) non-regular

(iv) regular

(d) The minimum number of spanning trees in a connected graph with n nodes is

(i) $n-1$

(ii) $n/2$

(iii) 2

(iv) 1

(3)

(e) If a set contains exactly m distinct elements where m denotes some non-negative integer, then the set is

(i) finite

(ii) infinite

(iii) None of the above

(iv) All of the above

(f) In an unweighted, undirected-connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of time complexity, by

(i) Dijkstra's algorithm starting from S

(ii) Warshall's algorithm

(iii) performing a DFS starting from S

(iv) performing a BFS starting from S

(g) The negation of 'Today is Friday' is

(i) Today is Saturday

(ii) Today is not Friday

(iii) Today is Thursday

(iv) Today is Sunday

(h) Whether the relation R on the set of all integers is reflexive, symmetric, anti-symmetric, or transitive, where $(x, y) \in R$ if and only if $xy \geq 1$?

(i) Anti-symmetric

(ii) Transitive

(iii) Symmetric

(iv) Both symmetric and transitive

(i) If p = It is raining and q = She will go to college. 'It is raining and she will not go to college' will be denoted by

(i) $p \wedge \neg q$

(ii) $p \wedge q$

(iii) $\neg p \wedge q$

(iv) $\neg (p \wedge q)$

(j) If $(x > 0$ and $x^2 < 0)$, then $x \geq 10$.

(i) True

(ii) False

(iii) Both (i) and (ii)

(iv) None of the above

2. (a) Define the following terms and give an example for each :

Reflexive relation, irreflexive relation, antisymmetric relation, partition set

(b) Prove that, if $S = \{s_1, \ldots, s_k\}$ be a set, then $P(S)$ has 2^k elements. 8+6

3. Define 'group', 'order of a group', and 'Abelian group'. Prove that every subgroup of a cyclic group is cyclic. 6+8

4. (a) Define the following with example :

Ring, homomorphism, cyclic group, coset

(b) Determine whether f is one-one or onto for the following cases .

(i) Let $A = B = \{1\ 2\ 3\ 4\}$ and $f = \{(1\ 1)(2\ 3)(4\ 2)\}$

(ii) Let $A = \{abc\}, B = \{1\ 2\ 3\ 4\}$ and $f = \{(a\ 1)(b\ 1)(c\ 4)\}$ 10+4

5. (a) Suppose that H and K are normal subgroups of G with $H \cap K = \{1\}$, show that $xy = yx$ for all $x \in H$ and $y \in K$. Let $I \in R$ be an ideal.

(b) The radical $\sqrt{I} = \{r \in R \mid rn \in I, n \in N\}$. Show that $\sqrt{I}$ is an ideal. 7+7

6. (a) State and prove De Morgan's laws of set theory.

(b) In a survey of 260 college students, the following data were obtained :

64 had taken a mathematics course, 94 had taken a computer science course, 58 had taken a business course, 28 had taken both a mathematics and a business course, 26 had taken both a mathematics and a computer science course, 22 had taken both a computer science and a business course, and 14 had taken all three types of courses.

(i) How many of these students had taken none of the three courses?

(ii) How many had taken only a computer science course? 4+10

7. (a) Prove that for any non-empty binary tree T, if n_0 is the number of leaves and n_2 be the number of nodes having degree two, then $n_0 = n_2 + 1$.

(b) Derive total number of nodes of a binary tree having depth n. 10+4

8. (a) What is graph? Differentiate between directed and undirected graph.

(b) How many different ways can you represent a graph? Explain each of the representations by examples. What is complete graph?

(c) Prove that the sum of degrees of all vertices in a graph is always even. 4+5+5

9. Consider the graph given below :
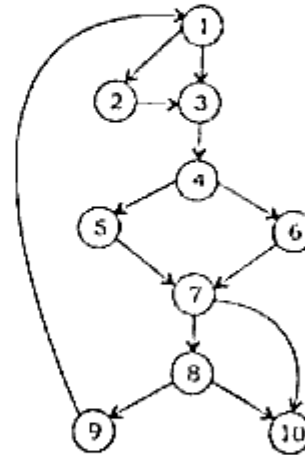
(a) Find the adjacency list and BFS traversal of the above graph.

(b) Prove that the maximum number of edges possible in a simple graph of n nodes is $n(n-1)/2$. 8+6

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