

Code : BSC-202 (100312)

(2)

B.Tech 3rd Semester Special
Exam., 2020

(New Course)

MATHEMATICS—III

(PDE, Probability and Statistics)

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
(ii) There are **NINE** questions in this paper.
(iii) Attempt **FIVE** questions in all.
(iv) Question No. 1 is compulsory.
(v) Relevant statistical data are given at the end of the question paper.

1. Choose the correct answer of the following
(any seven) : $2 \times 7 = 14$

(a) If P_n is the Legendre polynomial, then
the value of $\int_{-1}^1 x^m P_n dx$, ($m < n$) is

- (i) $\frac{2}{(2n+1)}$
 (ii) 0
(iii) 1
(iv) $\frac{2}{(2m+1)}$

(b) If J_n is the Bessel's function of first kind, then

$$\left[J_{-\frac{1}{2}}(x) \right]^2 + \left[J_{\frac{1}{2}}(x) \right]^2$$

is equal to

(i) $\sqrt{\frac{2}{\pi x}} \left(\frac{\cos x}{x} - \sin x \right)$

(ii) $\sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$

(iii) $\frac{2}{\pi x}$

(iv) $\frac{1}{\pi x}$

(c) The particular integral of
 $(D^2 - a^2 D'^2)Z = x^2$, is

(i) $\frac{1}{12} x^4$

(ii) $\frac{1}{3} x^3 + \frac{1}{2} yx^2$

(iii) $\frac{1}{4} x^4$

(iv) $x^4 + \frac{1}{2} yx^2$

(d) The function $3x^2 + 5x - 6$ in terms of Legendre polynomial is equal to

(i) $P_2 + 5P_1 - 5P_0$

(ii) $2P_2 + P_1 - 5P_0$

(iii) $2P_2 + P_1 - P_0$

(iv) $2P_2 + 5P_1 - 5P_0$

$$\frac{dy}{dx}$$

(e) Let the joint probability density function of the continuous random variables X and Y be

$$f(x, y) = \begin{cases} k(x^2 + y^2); & 0 < x < 1, 0 < y < 1 \\ 0 & ; \text{ elsewhere} \end{cases}$$

Then the value of k is

(i) 1

(ii) ~~3/2~~

(iii) 2

(iv) 5/2

(f) Let A and B be any two events. Which one of the following is correct?

(i) ~~$P(A \cap \bar{B}) = P(A) - P(A \cap B)$~~

(ii) $P(\bar{A} \cap B) = P(A) - P(A \cap B)$

(iii) $P(A \cup B) = P(A) + P(B)$

(iv) $P(A \cap B) = P(A) + P(B)$

(g) If $P(A \cap B) = \frac{1}{4}$, $P(A \cup B) = \frac{3}{4}$, $P(\bar{A}) = \frac{2}{3}$,

then $P(A/B)$ is equal to

(i) 1/3

(ii) 1/4

(iii) 1/2

(iv) 3/8

(h) If μ is the mean and σ is the standard deviation of a set of measurements, which are normally distributed, then the percentage of measurements within the range $\mu \pm \sigma$ is

(i) 70

(ii) 65

(iii) 67.45

(iv) 68.26

(i) If the density function of gamma distribution is

$$f(x) = \begin{cases} \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma \alpha}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Then mean is equal to

(i) α

(ii) β

(iii) $\alpha\beta$

(iv) $\alpha\beta^2$

- (j) The moment generating function of a continuous random variable X be given as $M_X(t) = (1-t)^{-5}$ for $|t| < 1$. Then its mean and variance are

(i) $\left(5, \frac{1}{5}\right)$

(ii) $\left(\frac{1}{5}, \frac{1}{5}\right)$

(iii) $\left(5, \frac{1}{15}\right)$

(iv) (5, 5)

2. (a) Solve :

$$(2xy-1)\frac{\partial z}{\partial x} + (z-2x^2)\frac{\partial z}{\partial y} = 2(x-yz)$$

- (b) Solve :

$$(D^2 - DD' - 2D'^2) Z = (2x^2 + xy - y^2)\sin xy - \cos xy$$

7+7

3. Reduce the following equation into canonical form and hence solve it : 14

$$x^2(y-1)\frac{\partial^2 z}{\partial x^2} + x(1-y^2)\frac{\partial^2 z}{\partial x \partial y} + y(y-1)\frac{\partial^2 z}{\partial y^2} + xy\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

4. State and prove orthogonal properties of Legendre polynomials. 14

5. (a) The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by this treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X , died. What is the chance that his disease was diagnosed correctly?

- (b) State axiomatic definition of the probability. Also prove that for any two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad 7+7$$

6. Let the joint probability density function of the continuous random variables X and Y be

$$f(x, y) = \begin{cases} kxy; & 0 < x < 2, 1 < y < 3 \\ 0; & \text{elsewhere} \end{cases}$$

- Find the value of k and probability density function of $X+Y$. Also find the mean and variance of X and Y . 14

7. (a) Fit a second degree parabola to the following data, where x is the independent variable :

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

(b) If X and Y are two uncorrelated variables and if $u = x + y$, $v = x - y$, then find the coefficient of correlation between u and v in terms of σ_x and σ_y the standard deviations of x and y respectively. 7+7

8. (a) The first four moments of a distribution about the value 4 of the variable are -1.5 , 17 , -30 , 108 . Find first four moments about the mean.

(b) In a distribution exactly normal, 17% of the items are under 35 and 79% items are under 63. What are the mean and standard deviation of the distribution? 7+7

9. (a) If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determine the probability that out of 1000 individuals (i) exactly 4 and (ii) more than 3 individuals will suffer a bad reaction.

(b) The mean lifetime of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean lifetime of all

the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against the alternative hypothesis $\mu \neq 1600$, using a level of significance of (i) 0.05 and (ii) 0.01. 7+7

Statistical data :

Given that $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}} dx$, while

$$\begin{aligned} f(0.81) &= 0.29, & f(0.96) &= 0.33, \\ f(1.0) &= 0.3413, & f(1.230) &= 0.39, \\ f(1.475) &= 0.43, & f(1.96) &= 0.4750, \\ f(2.0) &= 0.4772, & f(2.58) &= 0.4950, \\ f(3.0) &= 0.4987 \end{aligned}$$