Code : 101102

B.Tech 1st Semester Exam., 2019
(New Course)

MATHEMATICS—I

(Calculus, Multivariable Calculus and Linear Algebra)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Answer/Choose the correct option of the following (any *seven*) : 2×7=14

(a) At $x=a$, the function $f(x)$ defined as

$$f(x)=\begin{cases}\dfrac{x^2}{a}-a, & 0<x<a\\ 0, & x=a\\ a-\dfrac{a^3}{x^2}, & x>a\end{cases}$$

has

(i) continuity

(ii) mixed discontinuity

(iii) removable discontinuity

(iv) None of the above

(b) Write the statement of Maclaurin's theorem with remainders.

(c) In the expansion of $\log \sin x$ in power of $x-a$, the coefficient of $(x-a)^3$ is

(i) $2\operatorname{cosec}^2 a \cot a$

(ii) $\frac{1}{3}\operatorname{cosec}^2 a \cot a$

(iii) $\frac{2}{3}\operatorname{cosec}^2 a \cot a$

(iv) None of the above

(d) The function $e^x+2\cos x+e^{-x}$ has minima at $x=$

(i) π

(ii) $\frac{\pi}{2}$

(iii) 0

(iv) None of the above

(e) The radius of convergence of the power series

$$\sum \frac{(n!)^2 z^n}{(2n!)}$$

is

(i) 4

(ii) 1/4

(iii) 0

(iv) None of the above

(f) If the eigenvalue of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

is -3, then the eigenvalue of adj. A will be

(i) $-\frac{1}{3}$

(ii) $-\frac{1}{5}$

(iii) $-\frac{1}{15}$

(iv) -3

(g) Write down the quadratic forms corresponding to the given matrix

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 1 & 1 \end{bmatrix}$$

(h) The dimension of the vector space of all real numbers $\mathbb{R}$ over the field of rational numbers is

(i) 1

(ii) 2

(iii) 3

(iv) None of the above

(i) Which of the following sets of vectors is a basis for $\mathbb{R}^3$?

(1) $\{(1, 2, 3), (3, 5, 7), (5, 8, 11)\}$

(2) $\{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$

(3) $\{(1, 2, 3), (2, 3, 4), (2, 4, 6)\}$

(i) Only (1) and (2)

(ii) Only (2)

(iii) Only (1) and (3)

(iv) Only (1)

(j) Define range and kernel of linear map.

2. (a) State and prove the Lagrange's mean value theorem. 7

(b) Evaluate $\lim_{x \to 0} (a^x + x)^{\frac{1}{x}}$. 7

3. (a) Find the evolute of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ 7

(b) Expand $\tan x$ in power of $x - \frac{\pi}{4}$. 7

4. (a) Find the volume of the solid generated by revolving an arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ and x-axis about the x-axis. 7

(b) Evaluate the integral $\int_0^1 (1 - x^3)^{-1/2} dx$. 7

5. (a) Expand $f(x) = |\cos x|$ as Fourier series in $(-\pi, \pi)$. http://www.akubihar.com 7

(b) Show that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx = \frac{\beta(m, n)}{a^n (1+a)^m}$$ 7

6. (a) Find the rank of the matrix

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ 7

(b) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

and, hence, find the matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$ 7

7. (a) State and prove Cayley-Hamilton theorem. 7

(b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical forms. 7

8. (a) Let V be the set of all ordered (x, y), where x, y are real numbers. Let $a = (x_1, y_1)$ and $b = (x_2, y_2)$ be two elements in V. Define the addition as $a + b = (x_1, y_1) + (x_2, y_2) = (x_1 x_2, y_1 y_2)$ and the scalar multiplication as $\alpha(x_1, y_1) = (\alpha x_1, \alpha y_1)$. Check whether V is a vector space or not. Explain the region. 7

(b) Let U and V be two vector spaces in $\mathbb{R}^3$. Let $T: U \to V$ be a linear transformation defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + z \\ x + y \\ x + y + z \end{pmatrix}$$

Find the matrix representation of T with respect to the ordered basis

$$X = \left\{ \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$$

in U and

$$Y = \left\{ \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

in V. 7

9. (a) Let T be a linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$, where $Tx = Ax$, $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $x = (x\ y\ z)^T$. Find $\operatorname{Ker}(T)$, $\operatorname{ran}(T)$ and their dimensions. 8

(b) If (x, y, z) is a basis of $\mathbb{R}^3$ where $\mathbb{R}$ is the set of real numbers, then show that $(x+y, y+z, z+x)$ is also a basis of $\mathbb{R}^3$. 6

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