Code : 103102

B.Tech 1st Semester Exam., 2019
(New Course)

MATHEMATICS—I

(Calculus and Differential Equations)

Time : 3 hours

Full Marks : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are **NINE** questions in this paper.

(iii) Attempt **FIVE** questions in all.

(iv) Question No. **1** is compulsory.

1. Choose the correct answer of the following (any seven) : 2×7=14

(a) The minimum value of the function

$$f(x) = \sin x(1+\cos x),\ 0 < x < 2\pi$$

is

(i) π

(ii) $\frac{2\pi}{3}$

(iii) $\frac{5\pi}{3}$

(iv) 2π

(Turn Over)

(b) If

$$f(a+b-x) = f(x), \text{ then } \int_a^b x f(x)\,dx$$

is equal to

(i) $\left(\frac{a+b}{2}\right)\int_a^b f(b-x)\,dx$

(ii) $\left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$

(iii) $\left(\frac{b-a}{2}\right)\int_a^b f(x)\,dx$

(iv) $\left(\frac{a-b}{2}\right)\int_a^b f(x)\,dx$

(c) The slope of the tangent to the curve

$$y = \int_0^{x^2}\left(\frac{dt}{1+t^3}\right)$$

at the point where $x = 1$, is

(i) 2

(ii) 1

(iii) $\frac{1}{2}$

(iv) $\frac{1}{4}$

(d) The value of

$$\lim_{x\to 0} \frac{xe^{x^2}}{\int_0^x e^{t^2}\, dt}$$

is

(i) 0

(ii) 1

(iii) 2

(iv) −1

(e) The series whose nth term is

$$\{(n^3+1)^{\frac{1}{3}} - n\}$$

is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) None of the above

(f) The directional derivative of

$$\varphi(x, y, z) = x^2yz + 4xz^2$$

at the point (1, −2, −1) in the direction $2i - j - 2k$ is

(i) 1

(ii) 3

(iii) $\frac{11}{3}$

(iv) $\frac{37}{3}$

(Turn Over)

(g) The general solution of PDE

$$uu_x + yu_y = x$$

is

(i) $u^2 = g\left(\frac{y}{x+u}\right) + x^2$

(ii) $f(u^2 + x^2) = 0$

(iii) $f(x+y) = 0$

(iv) None of the above

(h) If J_n is the Bessel's function of first kind, then the value of $J_{\frac{3}{2}}$ is

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\sqrt{\frac{2}{\pi x}} \sin x$

(iv) $\sqrt{\frac{2}{\pi x}} \cos x$

(Continued)

(i) The general solution of

$$\frac{d^2y}{dx^2}+9y=\sin^3 x$$

is

(i) $y=A\cos(3x+B)+\frac{1}{24}\sin x-\sin 3x$

(ii) $y=Ae^{3x}+Be^{-3x}+\frac{1}{32}\sin x+\frac{1}{2}\cos 3x$

(iii) $y=A+Be^{3x}+2\sin x-\frac{5}{13}\sin 3x$

(iv) $y=A\sin(3x+B)+\frac{3}{32}\sin x+\frac{x}{24}\cos 3x$

(j) If P_n is the Legendre polynomial of first kind, then the value of

$$\int_{-1}^{1} P_{n+1}^2\, dx$$

is

(i) $\frac{2}{(2n+1)}$

(ii) $\frac{2}{(2n+2)}$

(iii) $\frac{2}{(2n+3)}$

(iv) $\frac{2}{(2n+4)}$

2. (a) Find the evolutes of the curve

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

where $a>b$.

(b) Prove that $\Gamma(1/2)=\sqrt{\pi}$.

7+7

3. (a) Find the extreme values of

$$f(x, y, z)=2x+3y+z$$

such that $x^2+y^2=5$ and $x+z=1$.

(b) Find θ, if

$$f(x+h)=f(x)+hf'(x)+\frac{h^2}{2!}f''(x+\theta h)$$

$0<\theta<1$ and $f(x)=ax^3+bx^2+cx+d$. 7+7

4. (a) Discuss the continuity of the function

$$f(x, y)=\begin{cases}\frac{x-y}{x+y}, & (x, y) \neq (0, 0)\\ 0, & (x, y) = (0, 0)\end{cases}$$

(b) Expand in the sine series of the function

$$f(x)=\begin{cases}x, & 0\le x\le 4\\ 8-x, & 4\le x\le 8\end{cases}$$

7+7

5. (a) Find the volume of the solid generated by revolving the ellipse $4x^2+9y^2=36$.

(b) Evaluate the integral by changing to polar coordinates

$$\int_0^1 \int_0^{\sqrt{(2x-x^2)}} (x^2+y^2)\,dy\,dx$$ 7+7

6. (a) Find the mass of a plate in the first quadrant of an ellipse $2x^2+3y^2=1$, whose density per unit area is given by $\rho=kxy$.

(b) Find the directional derivative of $\varphi(x, y, z)=x^2yz+4xz^2$ at $(1, -2, -1)$ in the direction $2i-j-2k$. 7+7

7. (a) Evaluate $\oint_C (xy)\,dx+(x^2+y^2)\,dy$, around the boundary of the region defined by $y^2=8x$ and $x=2$, using Green's theorem.

(b) Find $(\nabla\times A)\times B$, where

$$\vec{A}=x^2zi+yz^3j-3xyk \text{ and}$$
$$\vec{B}=3xi+4zj-xyk$$
7+7

(Turn Over)

8. (a) Solve the differential equation

$$\frac{d^2y}{dx^2}+9y=\sec 3t$$

by variation of parameters.

(b) Solve the differential equation

$$1+y^2+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0.$$ 7+7

9. (a) Prove that

$$\int_{-1}^{1} x^2 P_{n+1}P_{n-1}\,dx=\frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

(b) Find the complete integral of the partial differential equation

$$2xz+q^2=x(px+qy)$$ 5+9

...