(2)

Code: 102102/105102

## B.Tech 1st Semester Special Exam., 2020

( New Course )

MATHEMATICS-I

( Calculus and Linear Algebra )

Time: 3 hours

Full Marks: 70

Instructions:

The marks are indicated in the right-hand margin.

- There are **NINE** questions in this paper.
- (iii) Attempt FIVE questions in all.
- (iv) Question No. 1 is compulsory.
- 1. Choose the correct answer of the following (any seven):  $2 \times 7 = 14$ 
  - (a) If

$$Y = \int_0^\infty \frac{x^a}{a^x} dx, \ a > 1$$

then the value of Y is

(i) 
$$\frac{\Gamma(a)}{(\log_e a)^a}$$
 (ii)  $\frac{\Gamma(a+1)}{(\log_e a)^a}$ 

(ii) 
$$\frac{\Gamma(a+1)}{(\log_e a)^c}$$

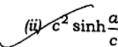
(iii) 
$$\frac{\Gamma(a+1)}{(\log_e a)^{a+1}}$$
 (iv) 
$$\frac{\Gamma(a)}{(\log_e a)^{a+1}}$$

$$\frac{\Gamma(a)}{(\log_e a)^{a+1}}$$

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The area bounded by the axis of x, and the curve and ordinates  $y = \cosh \frac{x}{x}$ from x = 0 to x = a is

(i) 
$$\cosh \frac{a}{c}$$



- (iii) csinh a
- (iv) None of the above
- Consider the following functions:

1. 
$$y = x \sin \frac{1}{x}$$
,  $x \ne 0$ ; and  $y = 0$  if  $x = 0$ 

2. 
$$y = x^2 \sin \frac{1}{x}$$
,  $x \ne 0$ ; and  $y = 0$  if  $x = 0$ 

3. 
$$y = x^2 \cos \frac{1}{x}$$
,  $x \ne 0$ ; and  $y = 0$  if  $x = 0$ 

4. 
$$y = x\cos\frac{1}{x}$$
,  $x \ne 0$ ; and  $y = 0$  if  $x = 0$ 

The functions, differentiable at x = 0, are

- (i) 1 and 2
- (ii) 2 and 3
- (iii) 3 and 4
- (iv) 1 and 4

- (d) For a positive term series  $\sum a_n$ , the ratio test states that
  - (i) the series converges, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}>1$$

(ii) the series converges, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}<1$$

(iii) the series converges, if

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=1$$

- (iv) None of the above
- (e) Ιf

$$\lim_{x\to\infty}\frac{\sin 2x + a\sin x}{x^3} = b$$

where b is finite, then the values of a and b respectively will be

- (ii) (2, 1)
- (iii) (-2, 1)
- (iv) (2, -1)

- The expansion of  $\tan x$  in powers of x by Maclaurin's theorem is valid in the interval
  - (i) (-00,00)

(ii) 
$$\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$$

(iii) 
$$(-\pi, \pi)$$
(iv)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ 

The value of

$$\lim_{\{x, y\}\to\{k, 0\}} \left(1 + \frac{x}{y}\right)^y$$

is

- (iv) Does not exist
- The gradient of the function  $f(x, y, z) = \sin(xyz)$ , at  $(1, -1, \pi)$ , is
  - $(\hat{i}), \pi(\hat{i}-\hat{j}+\hat{k})$
  - (ii)  $\pi(\hat{i} + \hat{j} + \hat{k})$
  - (iii)  $(\hat{i} + \hat{j} + \hat{k})$

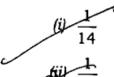
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 $f(v) = \{\mathbf{x}_i^2 \rightarrow \mathbf{x}_i^2 + \hat{\mathbf{x}}\}$ 

If det(A) = 7, where

$$A = \begin{bmatrix} a & b & c \\ 1 & 1 & g \\ g & \omega & 1 \end{bmatrix}$$

then  $det(2A)^{-1}$  is equal to



- (iii)  $\frac{1}{56}$
- (iv)  $\frac{7}{2}$
- If 3x+2y+z=0, x+4y+z=0 and 2x+u+4z=0 be a system of equations, then https://www.akubihar.com

fit is inconsistent

- (ii) it has only the trivial solution (0, 0, 0)
- (iii) it can be reduced to a single equation and so a solution does not exist
- the determinant of the matrix of coefficients is zero

Evaluate 2. (a)

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$$\int_0^\infty \log \left( x + \frac{1}{x} \right) \frac{dx}{1 + x^2}$$

- (b) Find the volume of the solid generated by rotating completely about the x-axis where the area enclosed between  $y^2 = x^3 + 5x$  and the line x = 2 and x = 4about its major axis.
- Find the maximum value of the function

$$f(x) = \frac{x}{1 + x \tan x}$$

- It is given that Rolle's theorem holds for  $f(x) = x^3 + bx^2 + cx,$ function  $1 \le x \le 2$  at the point  $x = \frac{4}{3}$ . Find the values of b and c.
- Discuss the convergence sequence whose n-th term is

$$a_n = \frac{(-1)^n}{n} + 1$$

(b) Test the convergence of the following series:

 $x^2 + \frac{2^2x^4}{3.4} + \frac{2^24^2x^6}{3.4.5.6} + \frac{2^24^26^2x^8}{3.4.5.6.7.8} \dots$ 

5. (a) Find the Fourier series expansion of the function  $f(x) = \{x^2, -2 \le x \le 2\}$ . Hence deduce that

 $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$ 

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(b) Find the Fourier cosine series and Fourier sine series of the following function in given interval:

 $f(x) = \begin{cases} x, & 0 < x < 2 \\ 2, & 2 \le x < 4 \end{cases}$ 

6/(a) Discuss continuity of the following function at the point (0, 0):

function at the point (0, 0):  $f(x, y) = \begin{cases} \frac{x^2y^2}{(x^3+y^3)}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ 

(b) Find the maximum value of xyz under the constraints  $x^2 + z^2 = 1$  and y - x = 0.

7. (a) Find the value of

 $\lim_{x\to\infty} \left(\frac{x+4}{x+2}\right)^{x+3}$ 

(b) Find the equation of the tangent plane to the surface  $x^2 - 3y^2 - z^2 = 2$ , at the point (3, 1, 2).

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8. Find the eigenvalues and eigenvectors of the following matrix :

 $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 

9 (a) Verify Cayley-Hamilton theorem for the matrix

 $\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ 

(b) Determine the range of the following linear: transformation. Also find the rank of T, where it exists.  $T: V_2 \rightarrow V_3$  defined by

 $T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$  7

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