Code : 102102/105102

B.Tech 1st Semester Special Exam., 2020

(New Course)

MATHEMATICS—I

(Calculus and Linear Algebra)

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are* ***NINE*** *questions in this paper.*

(iii) *Attempt* ***FIVE*** *questions in all.*

(iv) *Question No.* ***1*** *is compulsory.*

1. Choose the correct answer of the following (any *seven*) : 2×7=14

(a) If

$$Y = \int_0^\infty \frac{x^a}{a^x} dx, \; a > 1$$

then the value of Y is

(i) $\frac{\Gamma(a)}{(\log_e a)^a}$

(ii) $\frac{\Gamma(a+1)}{(\log_e a)^a}$

(iii) $\frac{\Gamma(a+1)}{(\log_e a)^{a+1}}$

(iv) $\frac{\Gamma(a)}{(\log_e a)^{a+1}}$

(b) The area bounded by the axis of x, and the curve and ordinates $y = c\cosh\frac{x}{c}$ from $x = 0$ to $x = a$ is

(i) $c\cosh\frac{a}{c}$

(ii) $c^2 \sinh\frac{a}{c}$

(iii) $c\sinh\frac{a}{c}$

(iv) None of the above

(c) Consider the following functions :

1. $y = x\sin\frac{1}{x}$, $x \neq 0$; and $y = 0$ if $x = 0$

2. $y = x^2 \sin\frac{1}{x}$, $x \neq 0$; and $y = 0$ if $x = 0$

3. $y = x^2 \cos\frac{1}{x}$, $x \neq 0$; and $y = 0$ if $x = 0$

4. $y = x\cos\frac{1}{x}$, $x \neq 0$; and $y = 0$ if $x = 0$

The functions, differentiable at $x = 0$, are

(i) 1 and 2

(ii) 2 and 3

(iii) 3 and 4

(iv) 1 and 4

(d) For a positive term series $\sum a_n$, the ratio test states that

(i) the series converges, if

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$$

(ii) the series converges, if

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$$

(iii) the series converges, if

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$$

(iv) None of the above

(e) If

$$\lim_{x\to\infty} \frac{\sin 2x + a\sin x}{x^3} = b$$

where b is finite, then the values of a and b respectively will be

(i) $(-2, -1)$

(ii) $(2, 1)$

(iii) $(-2, 1)$

(iv) $(2, -1)$

(f) The expansion of $\tan x$ in powers of x by Maclaurin's theorem is valid in the interval

(i) $(-\infty, \infty)$

(ii) $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

(iii) $(-\pi, \pi)$

(iv) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(g) The value of

$$\lim_{(x,\, y)\to(k,\, 0)} \left(1 + \frac{x}{y}\right)^y$$

is

(i) 1

(ii) e^{-k}

(iii) e^k

(iv) Does not exist

(h) The gradient of the function $f(x, y, z) = \sin(xyz)$, at $(1, -1, \pi)$, is

(i) $\pi(\hat{i} - \hat{j} + \hat{k})$

(ii) $\pi(\hat{i} + \hat{j} + \hat{k})$

(iii) $(\hat{i} + \hat{j} + \hat{k})$

(iv) $(\pi\hat{i} - \pi\hat{j} + \hat{k})$

(i) If $\det(A) = 7$, where

$$A = \begin{bmatrix} a & b & c \\ 1 & 1 & g \\ g & \omega & 1 \end{bmatrix}$$

then $\det(2A)^{-1}$ is equal to

(i) $\frac{1}{14}$

(ii) $\frac{1}{49}$

(iii) $\frac{1}{56}$

(iv) $\frac{7}{2}$

(j) If $3x + 2y + z = 0$, $x + 4y + z = 0$ and $2x + y + 4z = 0$ be a system of equations, then

(i) it is inconsistent

(ii) it has only the trivial solution (0, 0, 0)

(iii) it can be reduced to a single equation and so a solution does not exist

(iv) the determinant of the matrix of coefficients is zero

2. *(a)* Evaluate

$$\int_0^\infty \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2}$$ 7

(b) Find the volume of the solid generated by rotating completely about the x-axis where the area enclosed between $y^2 = x^3 + 5x$ and the line $x = 2$ and $x = 4$ about its major axis. 7

3. *(a)* Find the maximum value of the function

$$f(x) = \frac{x}{1 + x \tan x}$$ 5

(b) It is given that Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + cx$, $1 \le x \le 2$ at the point $x = \frac{4}{3}$. Find the values of b and c. 9

4. *(a)* Discuss the convergence of the sequence whose n-th term is

$$a_n = \frac{(-1)^n}{n} + 1$$ 7

(b) Test the convergence of the following series : 7

$$x^2 + \frac{2^2 x^4}{3.4} + \frac{2^2 4^2 x^6}{3.4.5.6} + \frac{2^2 4^2 6^2 x^8}{3.4.5.6.7.8} \cdots$$

5. (a) Find the Fourier series expansion of the function $f(x) = \{x^2, -2 \le x \le 2$. Hence deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$ 7

(b) Find the Fourier cosine series and Fourier sine series of the following function in given interval : 7

$$f(x) = \begin{cases} x, & 0 < x < 2 \\ 2, & 2 \le x < 4 \end{cases}$$

6. (a) Discuss continuity of the following function at the point (0, 0) : 7

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{(x^3 + y^3)}, & (x, y) \ne (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(b) Find the maximum value of xyz under the constraints $x^2 + z^2 = 1$ and $y - x = 0$. 7

7. (a) Find the value of

$$\lim_{x \to \infty} \left(\frac{x+4}{x+2} \right)^{x+3}$$ 4

(b) Find the equation of the tangent plane to the surface $x^2 - 3y^2 - z^2 = 2$, at the point (3, 1, 2). 10

8. Find the eigenvalues and eigenvectors of the following matrix : 14

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

9. (a) Verify Cayley-Hamilton theorem for the matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

(b) Determine the range of the following linear transformation. Also find the rank of T, where it exists. $T : V_2 \to V_3$ defined by

$$T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$$ 7

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